MATH 124, SAMPLE PROBLEMS FOR EXAM 2, 18 OCT, 2004
(Expect six problems, each worth 10 points.)

(1) Find all two-element generating sets for $S_3$. Pick one of them and draw the resulting Cayley digraph.

(2) Decide, with justification, whether or not $Z_6$ is isomorphic to $S_3$.

(3) Using decompositions into disjoint cycles and into transpositions, compute the order and the parity of the following element of $S_9$. What are the orbits of this permutation?

$$
\left(\begin{array}{cccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
3 & 6 & 4 & 5 & 1 & 8 & 9 & 2 & 7 \\
\end{array}\right)
$$

(4) Define the centralizer of an element $a$ in a group $G$ to be the subgroup of $G$ consisting of all elements of $G$ that commute with $a$. What is the centralizer, then, of the 3-cycle $(1, 2, 3)$ in the group $S_3$?

(5) Given that

$$
\left(\begin{array}{cccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 9 \\
3 & 4 & 5 & 1 & 8 & y & z & 7 \\
\end{array}\right)
$$

is an even permutation in $S_9$, what are the possible values for $x$, $y$, and $z$? (Justify your answer.)

(6) Give an example of an element of $S_5$ that has order 6. Are there any elements of order 4? 10? 50? (Why/Why not.)

(7) Is $\mathbb{Z}_8 \times \mathbb{Z}_6$ cyclic? (Why/Why not.)

(8) List all abelian groups of order 144. Which of them is isomorphic to $\mathbb{Z}_{12} \times \mathbb{Z}_{12}$?

(9) In the following list of abelian groups, identify which ones are isomorphic to one another. $\mathbb{Z}_{120}, \mathbb{Z}_{40} \times \mathbb{Z}_3, \mathbb{Z}_{20} \times \mathbb{Z}_6, \mathbb{Z}_8 \times \mathbb{Z}_{15}, \mathbb{Z}_{24} \times \mathbb{Z}_5, \mathbb{Z}_{12} \times \mathbb{Z}_{10}, \mathbb{Z}_4 \times \mathbb{Z}_2 \times \mathbb{Z}_{15}, \mathbb{Z}_{30} \times \mathbb{Z}_2 \times \mathbb{Z}_2, \mathbb{Z}_8 \times \mathbb{Z}_3 \times \mathbb{Z}_5$.

(10) Illustrate Cayley’s Theorem by embedding $Z_4$ into a symmetric group. (Be explicit about what the images of 0, 1, 2, and 3 are under this embedding.)
(11) Answer the following true or false:

(a) If $G$ is a finite group and $n$ divides $|G|$, then $G$ necessarily has a subgroup of order $n$.

(b) If $G$ is a finite group and $H$ is a subgroup of $G$, then $|H|$ necessarily divides $|G|$.

(c) Every group of order 31 is cyclic.

(d) Every group of order 30 is cyclic.

(e) $A_5$ can be embedded as a subgroup of $S_4$.

(f) The number of left cosets of a subgroup equals the number of right cosets of that subgroup.

(g) If $H$ is a subgroup of $G$, then every left coset of $H$ is a right coset (and vice versa).