Marquette University
1999

COMPETITIVE SCHOLARSHIP EXAMINATION IN MATHEMATICS

Do not open this booklet until you are directed to do so.

1. Fill out completely the following information about yourself.

PRINT

<table>
<thead>
<tr>
<th>Last name</th>
<th>First name</th>
<th>Initial</th>
<th>Phone No.</th>
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ADDRESS

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<tr>
<th>Street address</th>
<th>City</th>
<th>State</th>
<th>Zip</th>
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</table>

Your high school: Name ____________________________ City __________________________

High School Counselor or Advisor: ____________________________

2. This examination consists of two parts. The time allowed for each will be approximately 60 minutes. Should you finish Part I early, you may proceed to Part II.

3. Part I consists of 20 objective-type questions. Each question has five possible answers marked: A., B., C., D., E. Only one answer is correct. You are to circle the letter corresponding to the correct response for as many problems as you can.

Example: If \( x = 5 \) and \( y = -2 \), then \( x + 4y \) is

\[ A. \quad -3 \quad B. \quad -2 \quad C. \quad -1 \quad D. \quad 0 \quad E. \quad +1. \]

4. Part II consists of 3 subjective-type questions. Show a summary of your work in this booklet for each question you attempt, whether or not you obtain a complete solution. Scratch paper is provided but be sure to show the essential steps of your work concisely in the space provided for each question. Only the work appearing in this booklet will be scored. You will be scored on your method of attack, ingenuity, insight, inventiveness, and logical developments as well as your solutions.

5. Pencils and scratch paper will be provided. No tables, rulers, compasses, protractors, slide rules, calculators, or other aids are permitted.

6. a. The scoring of questions in Part I has been devised to discourage random guessing and will be computed as follows:

\[
\text{(three times number correct)} - \text{(number wrong)}.\]

b. The scoring for the three questions in Part II will be 15, 13, and 12 for a total of 40 points. Partial credit will be given so it will be to your advantage to do as much as you are able to do on each question.

7. For the scoring committee. Do not write in the box below.

<table>
<thead>
<tr>
<th>Part I:</th>
<th>Part II:</th>
<th>Score on Part I: _________</th>
<th>Score on Part I: _________</th>
<th>Score on Part I: _________</th>
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<tbody>
<tr>
<td>No. Correct: _________</td>
<td>Score on 1: _________</td>
<td>Score on 1: _________</td>
<td>Score on Part II: _________</td>
<td>Score on 2: _________</td>
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<td>No. Wrong: _________</td>
<td>Score on 2: _________</td>
<td>Score on 3: _________</td>
<td>TOTAL: _________</td>
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PART I

1. If the digit 1 is placed after a two digit number whose tens’ digit is $t$, and units’ digit is $u$, the
new number is:
   
   (A) $10t + u + 1$
   
   (B) $100t + 10u + 1$
   
   (C) $1000t + 10u + 1$
   
   (D) $t + u + 1$
   
   (E) none of these answers

2. If 64 is divided into three parts-proportional to 2, 4 and 6, the smallest part is:
   
   (A) $5\frac{1}{3}$
   
   (B) 11
   
   (C) $10\frac{2}{3}$
   
   (D) 5
   
   (E) none of these answers

3. A car travels 120 miles from $A$ to $B$ at 30 miles per hour but returns the same distance at 40
miles per hour. The average speed for the round trip is closest to:
   
   (A) 33 mph
   
   (B) 34 mph
   
   (C) 35 mph
   
   (D) 36 mph
   
   (E) 37 mph

4. The points $(6, 12)$ and $(0, -6)$ are connected by a straight line. Another point on this line is:
   
   (A) $(3, 3)$
   
   (B) $(2, 1)$
   
   (C) $(7, 16)$
   
   (D) $(-1, -4)$
   
   (E) $(-3, -8)$
5. If \( y = f(x) = \frac{x + 2}{x - 1} \), then it is incorrect to say:

- (A) \( x = \frac{y + 2}{y - 1} \)
- (B) \( f(0) = -2 \)
- (C) \( f(1) = 0 \)
- (D) \( f(-2) = 0 \)
- (E) \( f(y) = x \)

6. The limit of \( \frac{x^2 - 1}{x - 1} \) as \( x \) approaches 1 as a limit is:

- (A) 0
- (B) indeterminate
- (C) \( x - 1 \)
- (D) 2
- (E) 1

7. The largest integer which divides the expression \( n^3 - n \), for all possible integral values of \( n \), is:

- (A) 2
- (B) 3
- (C) 4
- (D) 5
- (E) 6

8. Of the following statements, the one that is incorrect is:

- (A) Doubling the base of a given rectangle doubles the area.
- (B) Doubling the altitude of a triangle doubles the area.
- (C) Doubling the radius of a given circle doubles the area.
- (D) Doubling the divisor of a fraction and dividing its numerator by 2 changes the quotient.
- (E) Doubling a given quantity may make it less than it originally was.
9. For the simultaneous equations $2x - 3y = 8$

\[ 6y - 4x = 9: \]

(A) $x = 4, y = 0$

(B) $x = 0, y = \frac{3}{2}$

(C) $x = 0, y = 0$

(D) there is no solution

(E) there are an infinite number of solutions

10. If the radius of circle is increased 100%, the area is increased:

(A) 100%

(B) 200%

(C) 300%

(D) 400%

(E) by none of these

11. The factors of the expression $x^2 + y^2$ are:

(A) $(x + y)(x - y)$

(B) $(x + y)^2$

(C) $(x^2 + y^2)(x^2 + y^2)$

(D) $(x + iy)(x - iy)$

(E) none of these

12. The formula which express the relationship between $x$ and $y$ as shown in the accompanying table is:

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>100</td>
<td>90</td>
<td>70</td>
<td>40</td>
<td>0</td>
</tr>
</tbody>
</table>

(A) $y = 100 - 10x$

(B) $y = 100 - 5x^2$

(C) $y = 100 - 5x - 5x^2$

(D) $y = 20 - x - x^2$

(E) none of these
13. \( \log p + \log q = \log(p + q) \) only if:

(A) \( p = q = \text{zero} \)

(B) \( p = \frac{q^2}{1 - q} \)

(C) \( p = q = 1 \)

(D) \( p = \frac{q}{q - 1} \)

(E) \( p = \frac{q}{q + 1} \)

14. At 2:15 o’clock, the hour and minute hands of a clock form an angle of:

(A) 30°

(B) 5°

(C) 22\( \frac{1}{2} \)°

(D) 7\( \frac{1}{2} \)°

(E) 28°

15. Successive discounts of 10% and 20% are equivalent to a single discount

(A) 30%

(B) 15%

(C) 72%

(D) 28%

(E) none of these

16. A 25 foot ladder is placed against a vertical wall of a building. The foot of the ladder is 7 feet from the base of the building. If the top of the ladder slips 4 feet, then the foot of the ladder will slide:

(A) 9 ft.

(B) 15 ft.

(C) 5 ft.

(D) 8 ft.

(E) 4 ft.
17. The pressure \( P \) of wind on a sail varies jointly as the area \( A \) of the sail and the square of the velocity \( V \) of the wind. The pressure on a square foot is 1 pound when the velocity is 16 miles per hour. The velocity of the wind when the pressure on a square yard is 36 pounds is:

(A) \( 10\frac{2}{3} \) mph
(B) 96 mph
(C) 32 mph
(D) \( 1\frac{2}{3} \) mph
(E) 16 mph

18. If \( a^x = c^b = b \) and \( c^y = a^z = d \), then

(A) \( xy = qz \)
(B) \( \frac{x}{y} = \frac{q}{z} \)
(C) \( x + y = q + z \)
(D) \( x - y = q - z \)
(E) \( x^y = q^x \)

19. Given 12 points in a plane no three of which are collinear, the number of lines they determine is:

(A) 24
(B) 54
(C) 120
(D) 66
(E) none of these

20. The sides of a triangle are in the ratio 6:8:9. Then:

(A) the triangle is obtuse
(B) the angles are in the ratio 6 : 8 : 9
(C) the triangle is acute
(D) the angle opposite the largest side is double the angle opposite the smallest side
(E) none of these
PART II

1. Two teams, \( M \) and \( N \), play a series of 7 games to determine a championship. The series stops as soon as one team has won 4 games. The outcome of the series can be specified by listing the winners of the games played, in the order that they are played. So some of the different possible outcomes are:

   NNNN
   NMMNNN
   MNMNNN

Determine how many outcomes are possible:
2. Given a point \((a, b)\) with \(0 < b < a\), determine the minimum perimeter of a triangle with one vertex at \((a, b)\), one on the \(x\)-axis, and one on the line \(y = x\). You may assume that a triangle of minimum perimeter exists. \((13\ \text{POINTS})\)
3. Suppose that $x^4 - 10x^3 + 23x^2 - 10x + 1 = 0$.  

(a) Let $y = x + \frac{1}{x}$. Find an equation that must be satisfied by $y$.

(b) What are the roots of $x^4 - 10x^3 + 23x^2 - 10x + 1 = 0$?