Do not open this booklet until you are directed to do so.

1. Fill out completely the following information about yourself.

PRINT

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<tr>
<th>Last name</th>
<th>First name</th>
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<th>Phone No.</th>
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ADDRESS

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Your high school: Name ___________________________ City ___________________________

High School Counselor or Advisor: ____________________________________________

2. This examination consists of two parts. The time allowed for each will be approximately 60 minutes. Should you finish Part I early, you may proceed to Part II.

3. Part I consists of 20 objective-type questions. Each question has five possible answers marked: a., b., c., d., e. Only one answer is correct. You are to circle the letter corresponding to the correct response for as many problems as you can.

Example: If \( x = 5 \) and \( y = -2 \), then \( x + 4y \) is

\[
(A) \quad -3 \quad (B) \quad -2 \quad (C) \quad -1 \quad (D) \quad 0 \quad (E) \quad 1.
\]

4. Part II consists of 3 subjective-type questions. Show a summary of your work in this booklet for each question you attempt, whether or not you obtain a complete solution. Scratch paper is provided but be sure to show the essential steps of your work concisely in the space provided for each question. Only the work appearing in this booklet will be scored. You will be scored on your method of attack, ingenuity, insight, inventiveness, and logical developments as well as your solutions.

5. Pencils and scratch paper will be provided. No tables, rulers, compasses, protractors, slide rules, calculators, or other aids are permitted.

6. a. The scoring of questions in Part I has been devise to discourage random guessing and will be computed as follows:

\[
\text{(three times number correct)} - \text{(number wrong)}.
\]

b. The scoring for the three questions in Part II will be: 10 points, 15 points, and 15 points respectively. Partial credit will be given so it will be to your advantage to do as much as you are able to do on each question.

7. For the scoring committee. Do not write in this space.

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<thead>
<tr>
<th>Part I:</th>
<th>Part II:</th>
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<tr>
<td>No. Correct:</td>
<td>Score on 1:</td>
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<tr>
<td>No. Wrong:</td>
<td>Score on 2:</td>
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<td>Score on 3:</td>
<td>Total:</td>
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PART I

1. If the sum $1 + 2 + 3 + \ldots + K$ is a perfect square $N^2$ and if $N$ is less than 100, then the possible values for $K$ are:

   (A) only 1
   (B) 1 and 8 only
   (C) only 8
   (D) 8 and 49 only
   (E) 1, 8, and 49

2. Four boys bought a boat for $60. The first boy paid one half of the sum of the amounts paid by the other boys; the second boy paid one third of the sum of the amounts paid by the other boys and the third boy paid one fourth of the sum of the amounts paid by the other boys. How much did the fourth boy pay?

   (A) $10
   (B) $12
   (C) $13
   (D) $14
   (E) $15

3. The product of $\sqrt{4}$ and $\sqrt{8}$ equals

   (A) $\sqrt{12}$
   (B) $2\sqrt{12}$
   (C) $\sqrt{32}$
   (D) $\sqrt[4]{32}$
   (E) $2\sqrt[4]{32}$

4. If $b > 1$, $x > 0$ and $(3c)^{\log_{3}x} - (3x)^{\log_{3}3} = 0$, then $x$ is

   (A) $\frac{1}{216}$
   (B) $\frac{1}{6}$
   (C) 1
   (D) 6
   (E) Not uniquely determined
5. A woman, her brother, her son and her daughter are chess players (all relations by birth). The worst player's twin (who is one of the four players) and the best player are of opposite sex. The worst player and the best player are of the same age. Who is the worst player?

(A) the woman
(B) her son
(C) her brother
(D) her daughter
(E) No solution is consistent with the given information.

6. Suppose you have proved statements I and II

I. If $P$, then $q$
II. If $S$, then not $q$.

Which statement follows from statements I and II?

(A) if $P$, then $S$
(B) if not $P$, then not $q$
(C) if $P$ or $q$, then $S$
(D) if $S$, then not $P$
(E) if not $S$, then $P$

7. In the expansion of $\left(a - \frac{1}{\sqrt{a}}\right)^7$ the coefficient of $a^{-\frac{1}{2}}$ is:

-7
(B) 7
(C) -21
(D) 21
(E) 35

8. The vertex of the parabola $y = x^2 - 8x + b$ will be a point on the $x$-axis if the value of $b$ is:

(A) -16
(B) -4
(C) 4
(D) 8
(E) 16
9. Given the line $3x + 5y = 15$. Let $P$ be a point on this line that is equidistant from the $x$-axis and the $y$-axis. Then $P$ can lie in:

(A) None of the quadrants
(B) quadrant I only
(C) quadrants I, II only
(D) quadrants I, II, and III only
(E) None of the above

10. Two swimmers, at opposite ends of a 90-foot pool, start to swim the length of the pool, one at the rate of 3 feet per second, the other at 2 feet per second. They swim back and forth for 12 minutes. Allowing no loss of time at the turns, find the number of times they pass each other.

(A) 24
(B) 21
(C) 20
(D) 19
(E) 18

11. The expression $1 - \frac{1}{1 + \sqrt{3}} + \frac{1}{1 - \sqrt{3}}$ equals

(A) $1 - \sqrt{3}$
(B) 1
(C) $-\sqrt{3}$
(D) $\sqrt{3}$
(E) $1 + \sqrt{3}$

12. In the equation $2x^2 - hx + 2k = 0$, the sum of the roots is 4 and the product of the roots is $-3$. Then $h$ and $k$ have the values, respectively:

(A) 8 and $-6$
(B) 4 and $-3$
(C) $-3$ and 4
(D) $-3$ and 8
(E) 8 and $-3$
13. For \( x \geq 0 \), the smallest value of \( \frac{4x^2 + 8x + 13}{6(x + 1)} \) is:

(A) 1  
(B) 2  
(C) \( \frac{25}{12} \)  
(D) \( \frac{13}{6} \)  
(E) \( \frac{34}{5} \)

14. In the figure below, \( AB \) is parallel to \( CD \), which of the following statements is true?

(A) \( \angle A + \angle E + \angle C = 180^\circ \)  
(B) \( \angle A + \angle E + \angle C = 270^\circ \)  
(C) \( \angle A + \angle E + \angle C = 360^\circ \)  
(D) \( \angle A + \angle E + \angle C = 400^\circ \)  
(E) None of above

15. Two circles with centers \( P \) and \( Q \) intersect at \( R \) and \( S \). Which of the following statements is not always true:

(A) \( PRQS \) has two sides of equal length.  
(B) \( PRQS \) has two angles of equal measure.  
(C) The line segments \( PQ \) and \( RS \) are perpendicular.  
(D) Angles \( SPR \) and \( SQR \) have the same measure.  
(E) Segments \( PS \) and \( PR \) have the same length

16. If \( x^4 + 4x^3 + 6px^2 + 4qx + r \) is exactly divisible by \( x^3 + 3x^2 + 9x + 3 \), the value of \( (p + q)r \) is:

(A) -18  
(B) 12  
(C) 15  
(D) 27  
(E) 45
17. At 2:15 o'clock, the hour and minutes hands of a clock form an angle of:

(A) 30°
(B) 5°
(C) 22 1/2°
(D) 7 1/2°
(E) 28°

18. A quadrilateral is inscribed in a circle. If angles are inscribed in the four arcs cut off by the sides of the quadrilateral, without intersecting the sides between vertices, their sum will be:

(A) 180°
(B) 540°
(C) 360°
(D) 450°
(E) 1080°

19. When simplified \(\sqrt{1 + \left(\frac{x^4 - 1}{2x^2}\right)^2}\) equals:

(A) \(\frac{x^4 + 2x^2 - 1}{2x^2}\)
(B) \(\frac{x^4 - 1}{2x^2}\)
(C) \(\frac{\sqrt{x^2 + 1}}{2}\)
(D) \(\frac{x^2}{\sqrt{2}}\)
(E) \(\frac{x^2}{2} + \frac{1}{2x^2}\)

20. In the figure, it is given that angle \(C = 90°\), \(\overline{AD} = \overline{DB}\), \(DE \perp AB\), \(\overline{AB} = 20\), and \(\overline{AC} = 12\). The area of quadrilateral \(ADEC\) is:

(A) 75
(B) 58 1/2
(C) 48
(D) 37 1/2
(E) None of these
PART II

1. A birthday cake is divided up between 4 guests at a party as follows. (10 points)
   
   • The first guest takes $\frac{1}{4}$ of the cake.
   • The second guest takes $\frac{1}{5}$ of the remainder.
   • The third guest takes $\frac{1}{16}$ of the remainder.
   • The fourth guest takes $\frac{1}{25}$ of the remainder.

   (i) What fraction of the cake is left after all 4 guests eat?

   (ii) Suppose that additional guests arrive and the $K^{th}$ guest takes a fraction $\frac{1}{(K+1)^2}$ of the part of the cake that is left when he arrives. How much cake is left after the 20$^{th}$ guest eats?

   (iii) If additional guests continue to arrive forever, how much cake will be left at the end of time?
2. The number 1995 has the special property that 199 (the number formed by the first 3 digits) exactly divides into 995 (the number formed from the last 3 digits). Since $995 = 199 \times 5$. Find all other 4 digit numbers with this property.
3. A distant planet, sometime in the distant future, has a network of one way highways that connect its cities. Not all pairs of cities are connected, but from every city it is possible to follow a sequence of highways to reach the capital, Rome. For example, the highways might be arranged as in the diagram

A Klingon general challenges a Romulan general to the following game. Each player in turn selects a city and instructs his armies to destroy that city and all cities that can be reached by highway from it. The player who is able to select the last city wins.

(i) Show that if the cities are arranged as in the diagram, then the player who selects first can guarantee to win the game.

(ii) Show that for any number of cities and any arrangement of highways, the player who selects first can guarantee to win the game (so long as Rome can be reached by a sequence of highways starting from any given city).