HW2

Assigned Jan 27, Thursday
Due 11:59PM Thursday Feb 3
Total: 22pts

You can work on this homework in a team of two students. Write down the team members’ names in the paper and the program. If your group collaborate with other teams, indicate the teams.

1. (4pts) Is $2^{n+1} = O(2^n)$? Is $2^{2n} = O(2^n)$? Prove your answers. Half of the points will be taken off if no proof is given.

2. (4pts) Rank the following functions by increasing order of growth. All the logs are in base 2.
   $N, \sqrt{N}, N^{1.5}, N^2, N \log N, N(\log N)^2, N \log N^2, 2/N, 2^N, 2^{N/2}, 37, N^2 \log N, N^3$. Indicate which functions grow at the same rate.

3. (4pts) Evaluate the following sums:
   - $\sum_{i=0}^{\infty} \frac{1}{4^i}$
   - $\sum_{i=0}^{\infty} \frac{i}{4^i}$

4. (10pts) A prime number has no factors besides 1 and itself. Do the following.
   - (2pts) Write a program to implement sieve of Eratosthenes algorithm that is introduced in page 7.
   - (1pt) For the input $N$ to the algorithm, what is the running time of the program?
   - (2pts) From scratch, write a method in the same program to determine $B$: the number of bits in the binary representation of $N$. You can only use +, -, *, /, mod operations in your program. You can’t invoke any of the Java methods that directly or indirectly calculate the bits.
• (2pts) Good program style (program usage, input checking, oo design principle) and formatting (documentation, indentation) count.

• (1pts) In terms of B, what is the worst-case running time of program?

• (2pts) Compare the running times to determine if a 20-bit number and a 40-bit number are prime theoretically and empirically.

Name your program file as "Prime.java". **Failure to follow the instructions on program name results in significant penalty.** "Prime.java" will have a main method that takes a number X as an argument, and prints:

The prime numbers under X are:

There are B bits in binary representation of X

To time your programs, you can use linux command "time". For example, the command

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  time java prime X
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will print out the execution time of program Prime. Note here, time prints the execution time of the entire program, not a single method. To time a single method, use JAVA System.currentTimeMillis() methods. Refer to [http://www.goldb.org/stopwatchjava.html](http://www.goldb.org/stopwatchjava.html) for details on how to use the method.

Turn in instructions:

Turn in your program Prime.java before class through turnin, and other parts in class Tuesday. When you turn in your programs, make sure they are for class cosc3100. You can specify the course in two ways: 1) If you run the turnin command without option "-c", you will be prompted to select one of the courses; 2) If you run "turnin -c cosc3100", that selects the course.