Unicoherency properties and inverse limits

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Outline

1. Co-existential Maps

2. Main Theorem

5. Open Problems
1. Co-existential Maps

The purpose of this talk is to announce some new preservation results concerning co-existential maps, as well as relate these results to preservation by inverse limits. by inverse limits that have continuous surjections for bonding maps.

Preliminary Definition: If $Y$ is a compactum, $I$ is a discrete set, and $\mathcal{D} \in \beta(I)$, let $p : Y \times I \to Y$ and $q : Y \times I \to I$ be the standard projection maps. Then the $\mathcal{D}$-ultracopower of $Y$ is the inverse image $\sum_\mathcal{D} Y$ of the point $\mathcal{D}$ under the Stone-Čech lift $q^\beta : \beta(X \times I) \to \beta(I)$, and the ultracopower projection map $p_{Y,\mathcal{D}} : \sum_\mathcal{D} Y \to Y$ is the restriction of $p^\beta : \beta(Y \times I) \to Y$ to the ultracopower.
Main Definition: A continuous surjection $f : X \to Y$ between compacta is a **co-existential map** if there is an ultracopower $\sum \mathcal{D}Y$ and a continuous surjection $g : \sum \mathcal{D}Y \to X$ such that $f \circ g = p_{Y,\mathcal{D}}$.

The notion of co-existential map mirrors that of existential embedding in model theory; nevertheless its behavior bears a strong resemblance to that of confluent map in continuum theory.
2. Main Theorem

Theorem 2.1: Let $G$ be a topological graph. Then co-existental maps preserve $G$-chainability (i.e., where every open cover refines to a finite open cover whose nerve is a polyhedron homeomorphic to $G$).

Corollary 2.2: Co-existental maps preserve chainability. They also preserve hereditary indecomposability; hence a co-existental image of a pseudo-arc is again a pseudo-arc.