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# CORRIGENDUM TO "TAXONOMIES OF MODEL-THEORETICALLY DEFINED TOPOLOGICAL PROPERTIES"

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**Abstract.** An error has been found in the cited paper; namely, Theorem 3.1 is false.

**1.** I would like to correct a simple, but serious, error in [1]; namely Theorem 3.1 therein is quite false: It *can* happen that there are compact Hausdorff spaces $X$ and $Y$ with $X \equiv Y$ (indeed $X \doteqdot Y$) but $X \not\equiv_{\mathbf{T}_t} Y$. I am most grateful to Lutz Heindorf for communicating [3] the following straightforward example: Let $X$ and $Y$ be any two Boolean spaces with infinite dense sets of isolated points. Then $B(X)$ and $B(Y)$ are Wallman bases for $X$ and $Y$ respectively, are infinite atomic Boolean algebras, and hence, by the Tarski invariants theorem, are elementarily equivalent. Thus $X \doteqdot Y$. However, one can easily pick $X$ and $Y$ as above so that $X \not\equiv_{\mathbf{T}_t} Y$; e.g., let $X$ and $Y$ be the ordinal spaces $\omega + 1$ and $\omega^2 + 1$ respectively. Then $Y$ has a point of Cantor-Bendixson derivative 2, while $X$ does not. This fact can be expressed in a sentence of $\Phi_t$.

The faulty inference in the proof of Theorem 3.1 of [1] occurs in the penultimate sentence: If $W$ and $Z$ are two Tichonov spaces with Wallman bases that are lattice-isomorphic, it does *not* generally follow that $W$ and $Z$ are homeomorphic. (We could make the inference if either $W$ and $Z$ were both compact or the Wallman bases contained all the singletons, but in our case $W$ and $Z$ are topological ultrapowers and neither condition holds.)

**2.** In Professor Heindorf's communication [3], there were some further interesting facts that enrich the content of [1].

**2.1.** The 3-cell $\mathscr{I}^3$ is characterized by $\mathbf{T}_F$ in {metrizable} [2]. (This augments Theorem 1.2 in [1].)

**2.2.** There is a complete description of the spaces that are (finitely) characterized by certain taxonomies in {metrizable Boolean}. Let $\mathscr{R}$ be the class of R. S. Pierce's "compact 0-dimensional metric spaces of finite type" [7].

THEOREM [5]. *For any metrizable Boolean space $X$, the following are equivalent*:

(i) $X \in \mathscr{R}$.

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(ii) *X is finitely characterized by* $\mathbf{T}_F$ *in* {*metrizable Boolean*}.

(iii) *X is finitely characterized by* $\mathbf{T}_t$ *in* {*metrizable Boolean*}. ∎

(This result addresses issue (**I2**) in [1].)

**2.3.** THEOREM [6]. *There are* $c$ $\mathbf{T}_t$*-taxa (hence* $c$ $\mathbf{T}_F$*-taxa) in* {*metrizable Boolean*}. ∎

(This result addresses issue (**I3**) in [1], and answers a question raised in the penultimate paragraph on p. 592 therein. See also the paragraphs following the proof of Theorem 2.10.)

**2.4.** THEOREM [4]. {*metrizable Boolean*} *is dense in* {*Boolean*}, *relative to* $\mathbf{T}_t$. ∎

(This result addresses issue (**I6**) in [1].)

## REFERENCES

[1] P. BANKSTON, *Taxonomies of model-theoretically defined topological properties*, this JOURNAL, vol. 55 (1990), pp. 589–603.

[2] H. G. BOTHE, *A first order characterization of 3-dimensional manifolds*, ***Workshop on extended model theory*** (H. Herre, editor), Report R-Math. 03/81, Akademie der Wissenschaften der DDR, Berlin, 1981, pp. 1–19.

[3] L. HEINDORF (private communication).

[4] ———, *Comparing the expressive power of various languages for Boolean algebras*, ***Zeitschrift für Mathematische Logik und Grundlagen der Mathematik***, vol. 27 (1981), pp. 419–434.

[5] ———, ***Beiträge zur Modelltheorie der Booleschen Algebren***, Seminarbericht No. 53, Humboldt-Universität, Berlin, 1984.

[6] B. MOLZAN, *On the number of different theories of Boolean algebras in several logics*, ***Workshop on extended model theory*** (H. Herre, editor), Report R-Math. 03/81, Akademie der Wissenschaften der DDR, Berlin, 1981, pp. 102–113.

[7] R. S. PIERCE, ***Compact zero-dimensional metric spaces of finite type***, Memoirs of the American Mathematical Society, no. 130 (1972).

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