(1) Let $f(x, y) = x + y$, with $R$ the triangle whose vertices are $(0, 0)$, $(0, 4)$, and $(4, 4)$.

(a) Approximate $\int_R f \, dA$ using the lower right-hand corners of the rectangles in the inner partition whose grid lines are given by $x = 0, 1, 2, 3, 4$ and $y = 0, 2, 4$.

(b) Set up and evaluate an iterated double integral that gives the exact value of $\int_R f \, dA$. 
(2) Let $f(x, y)$ be a continuous function defined on the region $R$, consisting of points in the first quadrant, bounded above by the curve $y = 4 - x^2$.

(a) Write $\int_R f \, dA$ as an iterated double integral, where $dA = dy \, dx$.

(b) Write $\int_R f \, dA$ as an iterated double integral, where $dA = dx \, dy$. 
(3) Let \( f(x, y, z) = x + z \), with \( W \) the solid hemisphere of radius 3, centered at the origin, \( z \geq 0 \).

(a) Express \( \int_W f \, dV \) as an iterated triple integral in \textit{cylindrical} coordinates.

(b) Express \( \int_W f \, dV \) as an iterated triple integral in \textit{spherical} coordinates.
(4) Let $C$ be the positively-oriented triangle that starts at the origin, goes in a straight line to $\langle 1, 3 \rangle$, then to $\langle 0, 3 \rangle$, then straight back to the origin.

(a) Evaluate $\int_{C_1} (x + y^2) \, dx + x^2 \, dy$, where $C_1$ is the first leg of the triangle $C$.

(b) Using Green’s Theorem, express the line integral $\int_C (x + y^2) \, dx + x^2 \, dy$ as an iterated double integral. (Don’t evaluate.)
(5) \( S \) is the upwardly-oriented graph of the function \( f(x, y) = 1 - x^2 \), sitting over the rectangle \( R = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 2\} \), and our vector field is given by \( \vec{F}(x, y, z) = xy\vec{i} + yz\vec{j} + xz\vec{k} \).

(a) If \( C = \partial S \), with orientation induced by \( S \), use Stokes’ theorem to express the line integral \( \int_C \vec{F} \cdot d\vec{r} \) as an iterated double integral. (Do not evaluate the integral.)

[Hint: \( dA = (-f_x\vec{i} - f_y\vec{j} + \vec{k}) \, dx \, dy \).]

(b) If \( W \) is the solid bounded by \( S \) on the top, \( R \) on the bottom, and the vertical planes \( x = 0, y = 0, y = 2 \) on the sides, use the divergence theorem to express the flux integral \( \int_{\partial W} \vec{F} \cdot d\vec{A} \) as an iterated triple integral.

(No evaluation necessary.)