Problem 6 (2.3(29)): Convert the following argument to symbols and decide whether it’s valid and what rule(s) of inference are used. If it isn’t valid, state where the error (i.e., converse or inverse) lies.

If at least one of these two numbers is divisible by six, then the product of these two numbers is divisible by six. Neither of these two numbers is divisible by six. Therefore the product of these two numbers is not divisible by six.

Let $P$ be “At least one of these two numbers is divisible by six,” and $Q$ be “The product of these two numbers is divisible by six.” Then the first sentence in the argument is $P \rightarrow Q$, the second sentence is $\sim P$, and the conclusion is $\sim Q$. This is a fallacious argument, exhibiting the inverse error: inferring $\sim Q$ from $P \rightarrow Q$ and $\sim P$.

Problem 7 (2.4(34e)): Write $P \leftrightarrow Q$ using Peirce arrows only.

We do the translation in easy stages, noting that $P \downarrow Q$ is the translation of $\sim (P \lor Q)$, and hence that $P \downarrow P$ is the translation of $\sim P$.

$P \leftrightarrow Q \equiv$

$(P \rightarrow Q) \land (Q \rightarrow P) \equiv$

$(\sim P \lor Q) \land (\sim Q \lor P) \equiv$

$I \sim (\sim P \lor Q) \land I \sim (\sim Q \lor P) \equiv$

$I \sim (\sim P \lor Q) \lor (\sim Q \lor P) \equiv$

$I \sim ((P \downarrow Q) \lor (Q \downarrow P)) \equiv$

$I \sim (P \downarrow Q) \land (Q \downarrow P \downarrow P)$.

Problem 8 (2.5(24)): Find the eight-bit two’s complement of 6710.

This will be the eight-bit representation of $2^8 - 67 = 189$, namely 10111101. (If we represent 67 in binary we get 1000011. Using eight-bit notation, we get 01000011; next if we flip 0s and 1s we get 10111100; finally, by adding 1, we obtain the two’s complement.)

Problem 9 (3.1(33d)): Let $\mathbb{R}$ be the domain for the individual variables $a, b, c, d$, with $<$ and $\cdot$ indicating the usual order and the usual multiplication of real numbers. Decide on the truth of the statement

$$[(a < b) \land (c < d)] \implies (a \cdot c < b \cdot d)$$

The use of the symbol $\implies$ is shorthand for saying “For all $a, b, c, d \ldots$.” This statement is not universally true because there is a counterexample: If $a = d = 0$, $b$ is positive, and $c$ is negative, then the hypothesis is true ($0 < b$ and $c < 0$) but the conclusion ($0 < 0$) is false.

Problem 10 (3.2(24b)): Rewrite the statements in symbolic format as implications and indicate the logical relationship between them.

All the integers that are greater than 5 and end in 1,3,7, or 9 are prime.

All the integers that are greater than 5 and are prime end in 1,3,7, or 9.
The two sentences may be viewed as quantified converses of one another if we choose the underlying domain $D$ to be all integers greater than 5. Let $E(x)$ say “$x$ ends in 1, 3, 7, or 9,” and let $P(x)$ say “$x$ is prime.” Then the two sentences above are respectively:

\[ \forall x \in D [E(x) \rightarrow P(x)] \]

\[ \forall x \in D [P(x) \rightarrow E(x)] \]