(Each of the following five problems is worth 12 points. Briefly justify all answers.)

(1) (a) A convex polygon has 30 vertices, and four faces meet at each vertex. How many faces are there?

(b) Use Dirac’s theorem to justify the fact that the octahedron graph is hamiltonian.

(2) (a) Give an example of a planar graph that is not hamiltonian.

(b) Draw $K_{3,3}$ in the plane, with a hamiltonian cycle bounding the unbounded region. Use the planarity algorithm for hamiltonian graphs to decide whether this graph is planar.
(3) (a) Show, by picture, that $K_{2,n}$ is planar for each $n \geq 1$.

(b) For which $n \geq 1$ is $K_{2,n}$ eulerian?

(4) (a) Show how a binary Gray code of order 3 may be viewed as a hamiltonian cycle in an appropriate graph.

(b) Show how a ternary memory wheel of order 2 may be obtained using an eulerian circuit in an appropriate graph.
(5) (a) In how many ways can a class of 16 engineering students be divided up into five working groups, one of size four and four of size three? (Write your answer in factorial notation.)

(b) What is the total number of partitions of a set of size four?