1. (15 pts)
   (a) Circle True or False.
   
   True       False   The function \( F(x) = \int_0^x \cos(t^2) \, dt \) is an antiderivative of \( f(x) = \cos(x^2) \).

   True       False   The integral \( \int_0^1 \frac{1}{x^2 - 4} \, dx \) is an improper integral.

   True       False   The geometric series \( \sum_{n=0}^{\infty} (-3)^n \) converges to \( \frac{1}{4} \).

   True       False   The function \( y = e^{2x} \) is a solution to the differential equation \( y'' = 4y \).

   (b) Give an example of an absolutely convergent series.

2. (10 pts) Use Integration by Substitution to evaluate the integral \( \int_0^{\pi/2} \cos(x)[5\sin(x) + 1]^3 \, dx \). SHOW ALL WORK.
3. (10 pts) Use Integration by Parts to evaluate the integral $\int xe^{2x} \, dx$. SHOW ALL WORK.

4. (15 pts) A ball is dropped from a 144 foot cliff.
   
   (a) Find the height of the ball, $h(t)$, in feet as a function of time. SHOW ALL WORK.
   
   (b) What is the ball’s velocity when it hits the ground? Do not forget units.
5. (10 pts) Determine whether the improper integral \( \int_{-\infty}^{-1} \frac{6}{x^4} \, dx \) converges or diverges. If it converges, calculate the integral. SHOW ALL WORK.

6. (15 pts) A rod of length 4 meters with density \( \delta(x) = 20 + x^2 \) kilograms/meter is positioned along the positive \( x \)-axis with its left end at the origin. Find the center of mass of the rod. SHOW ALL WORK, and do not forget units.
7. (15 pts)

(a) Sketch the solid obtained by revolving the region bounded by the curves \( y = \sqrt[3]{x}, \ y = 2, \) and \( x = 0 \) about the line \( y = -1. \)

(b) Set up the integral that gives the volume of the solid given in Part (a). DO NOT EVALUATE THE INTEGRAL.
8. (10 pts) An oil barrel is in the form of a right circular cylinder with height 3 feet and radius 1 foot. If the barrel is half filled with oil, set up the integral that gives the work required to pump all the oil out over the top of the barrel. The oil weighs 50 lbs/ft³. DO NOT EVALUATE THE INTEGRAL.

9. (10 pts) Use Limit Comparison Test or Comparison Test to determine whether \(\sum_{n=1}^{\infty} \frac{n^2}{5n^3 - 1}\) converges or diverges. State the test you use, and justify your answer.
10. (10 pts) Find the radius of convergence of the power series \( \sum_{n=1}^{\infty} \frac{5^n}{n} (x - 3)^n \). State any test you use, and justify your answer.
11. (10 pts) Find the Taylor series of \( f(x) = \frac{1}{x} \) about \( x = 3 \). Include an expression for the general term of the Taylor series. 

[Hint. Use fact \( f(x) = \frac{1}{x} \), \( f'(x) = -\frac{1}{x^2} \), \( f''(x) = \frac{2 \cdot 1}{x^3} \), \( f^{(n)}(x) = \frac{(-1)^n n!}{x^{n+1}} \).]

12. (10 pts) Using the Taylor series \( \sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + \frac{(-1)^n}{(2n+1)!} x^{2n+1} + \cdots \) for all \( x \), find the Taylor series for \( f(\theta) = \sin\left(\frac{\theta^2}{4}\right) \) about \( \theta = 0 \). Include an expression for the general term of the Taylor series.
13. (10 pts) Find the solution to the differential equation \( \frac{dy}{dx} = \frac{\sin(x)}{y^2} \) subject to initial condition \( y(0) = 4 \).