MATH 121, SAMPLE PROBLEMS FOR THE FINAL EXAM, 10 MAY, 2007

(Expect nine problems, each worth 10 points. The first four of them will be taken–almost *verbatim*–from the previous exams. The following are sample questions that cover new material.)

(1) Find an orthonormal basis for the orthogonal complement of $\text{span}\{[1, -1, 3]\}$ in $\mathbb{R}^3$.

(2) What are all possible values for x and y so that the matrix $\begin{bmatrix} 0.6 & x \\ 0.8 & y \end{bmatrix}$ is an orthogonal matrix?

(3) Find the projection matrix for the subspace $W = \text{span}\{[1, 1, 1], [1, -2, 1]\}$.

(4) Find the least squares solution to the system:

$$\begin{array}{rcrcr} x & + & 2y & = & 3 \\ x & + & y & = & 1 \\ 2x & + & 3y & = & 3 \end{array}$$

(5) Find the quadratic function that best fits the data $\{(0, 0), (1, 3), (2, 4), (3, 7)\}$, in the least squares sense.

(6) Answer true or false:

(a) The image of a projection matrix is an eigenspace for that matrix.

(b) Every projection matrix is an orthogonal matrix.

(c) The inverse of a projection matrix is a projection matrix.

(d) The inverse of an orthogonal square matrix is orthogonal.

(e) A nonsingular projection matrix is the identity matrix.

(f) An orthogonal linear transformation is angle preserving.

(g) An angle-preserving linear transformation is orthogonal.

(h) If the columns of A and the columns of B are bases for the same subspace of $\mathbb{R}^n$, then $A(A^\top A)^{-1}A^\top = B(B^\top B)^{-1}B^\top$.

(i) If the least squares solution to $A\vec{x} = \vec{b}$ is an actual solution, then the coefficient matrix must be square.

(7) Show that an invertible idempotent matrix is the identity matrix.

(8) Show that the product of two orthogonal square matrices is orthogonal.