MATH 121, SAMPLE PROBLEMS FOR EXAM 2, 05 MARCH, 2007

(Expect six problems, each worth 10 points.)

(1) Find the rank, as well as a basis for the column space of, the matrix:

$$A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 0 & 4 & 2 \\ 3 & 2 & 8 & 7 \end{bmatrix}$$

(2) Enlarge $\{[2,1,1],[1,0,1]\}$ to a basis for $\mathbb{R}^3$.

(3) Answer true or false:

(a) In any matrix, the number of independent row vectors equals the number of independent column vectors.

(b) The rank of an invertible square matrix equals the number of rows.

(c) No matrix has rank zero.

(d) The zero vector may fail to be in the range of a linear transformation.

(e) A linear transformation is determined by where it sends the vectors in a basis.

(f) Distinct vectors in a finite-dimensional vector space V have distinct coordinate vectors relative to a given ordered basis B for V.

(g) The vector space P_8 of polynomials of degree ≤ 8 is isomorphic to $\mathbb{R}^8$.

(4) Suppose $T : \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation taking $[-1,2]$ to $[1,0,0]$ and $[2,1]$ to $[0,1,2]$. Find a general rule for computing $T([x,y])$, and compute $T([3,1])$.

(5) Suppose $T : \mathbb{R}^3 \to \mathbb{R}^3$ is defined by the rule $T([x,y,z]) := [2x+y+z, x+z, y]$. Find the standard matrix representation for T.

(6) Decide (with justification) whether the set $\mathbb{R}^2$, with usual vector addition and scalar multiplication defined by $r[x,y] := [ry, rx]$, is a vector space.

(7) Show that the set D of all diagonal 2×2 matrices–i.e., zeros off the main diagonal–is a subspace of the space M_2 of all 2×2 matrices.

(8) Show that, for each natural number n, the set $\{1, x, x^2, x^3, ..., x^n\}$ is not a basis for the vector space $\mathbb{R}(x)$ of all polynomials in the indeterminate x.

(9) Prove that $\{1, \sin x, \sin 2x\}$ is an independent set of functions in the vector space $\mathbb{R}^{\mathbb{R}}$ of all functions from $\mathbb{R}$ to $\mathbb{R}$.

(10) Find the polynomial in P_2 whose coordinate vector relative to the ordered basis $(x^2 + 1, x^2 - 1, x)$ is $[2, 5, -1]$.

(11) Let $T : P_2 \to P_2$ be differentiation. Find the matrix representation for T relative to the ordered basis in Problem 10.

(12) Let C be the vector space of all continuous functions from $\mathbb{R}$ to $\mathbb{R}$, and let $T : C \to \mathbb{R}$ be defined by the definite integral as follows: $T(f) := \int_0^1 f(x)\, dx$. Find two distinct vectors in the kernel of T.