MATH 121, SAMPLE PROBLEMS FOR EXAM 1, 09 FEB, 2007

(Expect six problems, each worth 10 points.)

(1) Find a unit vector parallel to $[5, 0, -12]$.

(2) Find a vector in $\mathbb{R}^3$ that is perpendicular to both $[1, 1, 1]$ and $[1, 0, 1]$.

(3) Answer the following statements true or false, where all matrices are square, of the same dimensions:

(a) If A is invertible, then so is its transpose, A^T.

(b) If $AC = BC$ for some matrix C, then $A = B$.

(c) If $AB = BA$, then $A = B$.

(d) If A is symmetric, then so is A^T.

(e) If A and B are symmetric, then so is AB.

(4) Compute the *reduced* row echelon form H for the matrix A below, and write H in the form BA, where B is a product of elementary matrices.

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(5) Express as a parameterized family of vectors, the solution set for the system:

$$\begin{array}{ccccccccc} x_1 & & & + & 2x_3 & & & = & 1 \\ & & x_2 & & & + & 3x_4 & = & 2 \end{array}$$

(6) Show that $[1, 2]$ is in the span of the set $\{[1, 3], [2, -6]\}$.

(7) Find a 2×2 non-invertible matrix A with no zero entries.

(8) Determine (with justification) whether the set of column vectors of A is a basis for the column space of A, where A is the matrix:

$$\begin{bmatrix} 2 & 1 & 3 \\ 1 & 4 & 0 \end{bmatrix}$$

(9) Find a basis for the null space of the matrix:

$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 3 \end{bmatrix}$$