1

Given $f(n) = 4n \lg n + n$ and $g(n) = \frac{n^2 - n}{2}$, determine which one is true: $f(n) \in O(\) or $g(n) \in O(f(n))$.

2

Given $f(n) = n + n\sqrt{n}$ and $g(n) = 4n \lg (n^2 + 1)$, determine which one is true: $f(n) \in O(\) or $f(n) \in \Omega(\) or $f(n) \in \Theta(\).

3

The range of a finite nonempty set of $n$ real numbers $S$ is defined as the difference between the largest and smallest elements of $S$. For each representation of $S$ given below, describe in English an algorithm to compute the range. Indicate the time efficiency classes of these algorithms using the most appropriate notation ($O$, $\Theta$, or $\Omega$).

1. An unsorted array
2. A sorted array
3. An array which is a concatenation of two sorted portions.
4. A sorted singly linked list
5. A binary search tree
What is the value returned by the following algorithm? Express your answer as a function of \( n \). Give, using \( O \)-notation, the worst-case running time. [Hint: You could verify whether your answer is correct by writing a program that implements this algorithm and see what \( n \) gives what \( r \).]

Algorithm 1: Secret\((n)\)
\[
\begin{aligned}
    & r \gets 0 \\
    & \text{for } i \gets 1 \text{ to } n \text{ do} \\
    & \quad \text{for } j \gets i + 1 \text{ to } n \text{ do} \\
    & \quad \quad \text{for } k \gets i + j - 1 \text{ to } n \text{ do} \\
    & \quad \quad \quad r \gets r + 1 \\
    & \quad \end{aligned}
\]
return \( r \)

5
Design a recursive algorithm for computing \( 2^n \) for any nonnegative integer \( n \) that is based on the formula \( 2^n = 2^{n-1} + 2^{n-1} \). Set up a recurrence relation for the number of additions made by the algorithm and solve it. Draw a tree of recursive calls for this algorithm and count the number of calls made by the algorithm. Is it a good algorithm for solving this problem?

6 Bonus!
Prove that the recurrence relation in the pizza problem is correct (On slide #55 of Chapter 2 slides).