Time Series Data Mining (TSDM):
A New Temporal Pattern Identification
Method for Characterization and Prediction
of Complex Time Series Events

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An Ancient Chinese Say …

➢ You are not “grown-ups” till you reach the age of 30.
➢ You are not “free from doubts” till the age of 40.
➢ You do not know God’s Will till the age of 50.
➢ The bottom line:

Learning is a life-long process
Overview of Presentation

- Problem Statement
  - Graphical Problem Statement
  - Time Series Analysis Literature
  - Innovative New Approach
- Algorithm
  - Phase Space
  - Mathematical Formulation
  - Algorithm Results
- Applications
  - Progression of Time Series
  - Engineering
  - Financial

Collaborators/Credits

- Dr. Noveen Bansal, MSCS
- Dr. Richard Povinelli, EECE
- Hai Huang, Microsoft, Inc.
- Odilon K. Senyana, FAA
- Dr. Wenjing Zhang, Discover, Inc.
- Shaobo Wang, MU
Graphical Problem Statement

- Can we automatically characterize these sequences (temporal patterns)?
- Can we use such temporal patterns for prediction?

Temporal Pattern

- Find temporal patterns
- \( p \in P \subseteq \mathbb{R}^Q \), a vector of length \( Q \)

![Graph](image_url)
Events

- Temporal patterns that characterize and predict events
- Chosen a priori
  - Algorithm not restricted by event definition
  - Event definition is problem specific

![Graph showing time instances with high event value]

Time Series Analysis Literature

- Box-Jenkins, ARMA
  - Pandit and Wu (1983)
  - Bowerman and O’Connell (1993)

- Chaotic deterministic
  - Takens (1981)
  - Sauer (1991)
  - Casdagli (1989)
  - Abarbanel (1990, 1994)
  - Ghoshray (1996)
An Innovative Approach

- **Time Series Data Mining**
  - Time Series, \( X = \{x_t, t = 1, \ldots, N\} \)
  - Find temporal patterns that are characteristic and predictive of an event \( g(x_t) \)
  - \( p \in P \subseteq \mathbb{R}^Q \)
  - Nonstationary, non-periodic time series
  - Chaotic deterministic time series whose attractors are non-stationary

- **Local model, local prediction**
  - Not concerned with characterizing and predicting everywhere (every time)
  - Characterize and predict events

- **Applying Genetic Algorithm (GA) to find the “optimal” local model**

- **Exciting, new results with difficult real world time series**

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Introduction to Data Mining

- A step in the knowledge discovery process
- Application of algorithms to extract *meaningful* patterns

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Data Mining and Knowledge Discovery

- "The nontrivial extraction of implicit, previously unknown, and potentially useful information from data"[1]
- Uses artificial intelligence, statistical and visualization techniques to discover and present knowledge in a form which is easily comprehensible to humans.


More recently (already obsolete):

- “The huge amount of tracking data available from web sites as an "information gold mine."
  ➢ Business Week

- “74% of large companies expect to mine web data to increase profits by 2002.”
  ➢ Forrester Research (1998)
Popular Data Mining Problems

- Associating – Identifies patterns or groups of items
  
  “men who buy red ties also often buy cigars.”

- Classifying – Identifies clusters of items with common attributes
  
  “men who buy red ties and cigars also usually have wine at lunch and pay by credit card.”

- Sequencing – Identifies the order of events
  
  “men tend to buy red ties before lunch and cigars after lunch.”

- Predictive Modeling – Identifies a likely outcome from item clusters.
  
  “men who buy red ties and cigars and have wine at lunch are very likely to buy a silver Benz within two years and finance their purchase by borrowing money from bank.”
**Gold Mining Analogy**

- **Where do prospectors search for the gold?**
  - Geological formation
  - Quartz and ironstone
  - Structures such as banded iron formations
- **Data Mining**
  - Define formations that point to nuggets of information
  - Define patterns that identify an information strike
- **Definition of “gold”**
  - For Gold Mining
    - Size of nuggets makes a difference
    - Mining for oil or silver is different
  - Data Mining
    - Definition of knowledge
    - Clearly define the desired nuggets of information

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**Knowledge Discovery in Databases**

1. Cleaning Integration
2. Selection Transformation
3. Data Mining
4. Evaluation Visualization

Data → Data Warehouse → Prepared data → Patterns → Knowledge Base → Knowledge
Features of Data Mining

- Intend to discover knowledge which are previously unknown
- A Multi-disciplinary field growing out of many areas
  - Mathematical modeling and statistics
  - Pattern recognition
  - Computational intelligence

Data Mining Tools

- Traditional pattern recognition algorithms:
  - Data Visualization, Graphics
- Intelligent computing:
  - Artificial Neural networks, Fuzzy Logic, Genetic Algorithms/Evolutionary Computing, Expert Systems, Natural Language Processing, etc.

They all use “mechanical” computing power:

- Database Systems; Client-Server; Internet/WWW;
- Software/programming; etc.
Chaotic Deterministic Time Series

- No universal definition
- Characterized by the following criteria
  - Sensitivity to initial conditions
  - Positive Lyapunov exponent
  - Broadband Fourier spectra
  - Finite, possibly fractional attractors

Attractors

- Given a manifold \( M \) and a map \( f: M \to M \), an invariant set \( S \) is defined as follows
  \[
  S = \{ x_0; x_0 \in S, f^n(x_0) \in S, \forall n \} \subset M
  \]
- A positively invariant set requires that \( n \geq 0 \).
- A set \( A \subset M \) is an attracting set if \( A \) is a closed invariant set and there exists a neighborhood \( U \) of \( A \) such that \( U \) is a positively invariant set and
  \[
  f^n(x) \to A \quad \forall \ x \in U
  \]
- An attractor is defined as an attracting set that contains a dense orbit.

Gold Mining Analogy

- Where do prospectors search for the gold?
  - Geological formations
    - Quartz and ironstone
    - Structures such as banded iron formations
  - Data Mining
    - Define formations that point to nuggets of information (events)
    - Define temporal patterns that identify an information strike.

- Definition of gold
  - Size of nuggets makes a difference in how the mining is approached.
  - Mining for oil or silver is different
  - Data Mining
    - Definition of knowledge
    - Clearly define the desired nuggets of information (events)
    - Define event function \( g(x) \)

Two ideas arise from the analogy

- Patterns (Temporal Patterns)
  - Patterns that guide and direct to the nuggets of information need to be understood and identified
  - Temporal patterns and time series embedding

- Gold (Events)
  - Nuggets of information require clear definition
  - Time series events

- Relationship between a temporal pattern and event
  - Find “temporal patterns” that indicate a high event value
  - Example
    - A sequence of charge flow values in the brain that precede a physical response
    - A series of voltage values that precede the release of a droplet of metal from a welder
    - A sequence of stock prices that precede a rapid increase in the stock price
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Algorithm

- **Search for the optimal temporal pattern that yields the maximal “event” values**
- **Choose the support, \( Q \), of the temporal pattern \( p \)**
- **Define the event function, \( g \)**
- **Embed the time series into a phase space**
  - Define a metric to allow comparison between the temporal pattern and embedded time series
- **Find the “optimal” pattern cluster**
  - Pattern neighborhood that contains the maximum average “eventness”
  - Genetic Algorithm
Graphical Mapping

Phase Space with Event Values
Mathematical Formulation I

- **Non-stationary training time series**
  - $N$ is the length of the series
  
  $$X = \{x_t, t = 1, \ldots, N\}$$

- **Testing time series**
  
  $$Y = \{x_t, t = R, \ldots, S\}, \quad R > N$$

- **Define Event Function**
  - Inputs must be chosen carefully
  - Relationship to temporal pattern is important
  - Example $g$
    - The percentage change in the time series between times $t+Q$ and $t+Q+1$
    - $Q$ is the length of the temporal pattern
  
  $$g(x_t) = \frac{x_{t+Q} - x_{t+Q-1}}{x_{t+Q-1}}$$

Mathematical Formulation II

- **Define $P \subseteq \mathbb{R}^Q$**
  - A $Q$ dimensional real space
  - Phase space

- **Embed $X$ into $P$, likewise for $Y$ create $y_t$**
  - Subsets, $x_t$, of $X$ of cardinality $Q$
  - Chosen by any consistent rule
  
  $$x_t^T = (x_t, x_{t+\tau_1}, \ldots, x_{t+\tau_{Q-1}}), \quad t = 1, \ldots, N - \tau_{Q-1}$$

  - $\tau_1 < \tau_2 < \ldots < \tau_{Q-1}$

- **Define metric $d$ for $P$**
  - $d(p, x_t)$
  - Compare the embedded time series and the temporal pattern
  - $p$ - temporal pattern, a vector of length $Q$
  - $x_t$ - embedded time series
Property 1

- Find a temporal pattern \( p \in P \) and a threshold \( \delta \in \mathbb{R} \) that have the following two properties

- **Property 1**
  - Given the following definitions
    
    \[
    M_{\text{train}} = \{ t : d(p, x_t) \leq \delta \}, \quad t = 1, \ldots, N - Q - 1
    \]
    
    \[
    \mu_{M_{\text{train}}} = \frac{1}{c(M_{\text{train}})} \sum_{t \in M_{\text{train}}} g(x_t)
    \]
    
    \[
    \mu_X = \frac{1}{N - Q + 1} \sum_{i=1}^{N-Q+1} g(x_i)
    \]
  
  - That \( \mu_{M_{\text{train}}} > \mu_X \)
  
  - and the set \( \{ g(x_t) : t \in M_{\text{train}} \} \) is statistically different from the set \( \{ g(x_t) : t = 1, \ldots, N - Q + 1 \} \)

Property 2

- **Property 2**
  - Given the following definitions
    
    \[
    M_{\text{test}} = \{ t : d(p, y_t) \leq \delta \}, \quad t = R, \ldots, S - Q - 1
    \]
    
    \[
    \mu_{M_{\text{test}}} = \frac{1}{c(M_{\text{test}})} \sum_{t \in M_{\text{test}}} g(x_t)
    \]
    
    \[
    \mu_Y = \frac{1}{S - Q - R + 2} \sum_{t=R}^{S-Q+1} g(x_t)
    \]
  
  - That \( \mu_{M_{\text{test}}} > \mu_Y \)
  
  - and that the set \( \{ g(x_t) : t \in M_{\text{test}} \} \) is statistically different from the set \( \{ g(x_t) : t = R, \ldots, S - Q + 1 \} \)
Mathematical Formulation III

- Define threshold $\delta$
  - In terms of normalized threshold $\delta_d$
  - Use the mean and standard deviation statistics of $d(p, x_t)$

$$\mu_d = \frac{1}{N - \tau} \sum_{t=1}^{N-\tau} d(p, x_t)$$

$$\sigma_d^2 = \frac{1}{N - \tau} \sum_{t=1}^{N-\tau} (d(p, x_t) - \mu_d)^2$$

$$\delta = \mu_d + \delta_d \sigma_d$$

- Optimization formulation
  - Constraint forces cluster to be greater than 1 in size

$$\max_{\mathbf{p}, \mathbf{d}} f(\mathbf{p}, \mathbf{d}, X, g) = \mu_{M_{max}} = \frac{1}{c(M)} \sum_{i=1}^{M_{max}} g(x_i)$$

subject to $c(M_{train}) > \beta N$, $0 < \beta \leq 1$.
Time Series View of Results

Test Series Phase Space

next 100 points of nonstationary, non-periodic time series
Test Series Results

- Highest gold value time instances
- Pattern neighborhood from training
- Temporal pattern from training

Testing Phase Space

Event Values and Training Pattern Cluster
Time Series View of Testing

Pattern, $p$, is from training process

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- Proposed Work
Applications

- Set of progressively more complex time series
  - Show how each is embedded into the phase space
  - Show results of algorithm
  - Constant
  - Sinusoidal
  - Chirp
  - Random noise

- Engineering Application
  - Welding Time Series

- Financial Application
  - Stock Open Price

Progression of Time Series

- Choose the support of the pattern, $Q = 2$, to allow graphical presentation
- Define the gold function
  - The $t+Qth$ value in the time series
  - $Q$ is the length of the pattern

$$g(x_t) = B^{-(Q+1)} x_t$$
Constant Train Phase Space

Training portion of time series

Constant Test Phase Space

Testing portion of time series

Q = 2
Linearly Increasing Train

Training portion of time series

Q = 2

Linearly Increasing Test

Testing portion of time series

Q = 2
Sinusoidal Train Phase Space

Training portion of time series

Sinusoidal Test Phase Space

Testing portion of time series

Q = 2
Chirp Train Phase Space

Training portion of time series

Chirp Test Phase Space

Testing portion of time series
Noisy Sinusoidal Train Phase Space

Noisy Sinusoidal Test Phase Space
Noise Train Phase Space

Noise Test Phase Space

Uniform Density Random Variable

Testing portion of time series

Q = 2

Q = 2
Constant Pattern Cluster

![Graph of Constant Pattern Cluster]

Linearly Increasing Pattern Cluster

![Graph of Linearly Increasing Pattern Cluster]
Sinusoidal Pattern Cluster

Time Series View of Sinusoidal
Sinusoidal Statistical Tests

- **Runs Test**
  - $H_0$: There is no difference between the matched time series and the whole time series.
  - $H_A$: There is significant difference between the matched time series and the whole time series.
  - Using a 1% probability of Type I error ($\alpha = 0.01$).
  - $\alpha = 1.87 \times 10^{-18}$ which means the null hypothesis can easily be rejected.

- **Difference of two independent means**
  - Although the two populations are probably dependent, this can be ignored because it makes the statistics more conservative, i.e., it will tend to overestimate the Type I error.
  - $H_0$: $\mu_M - \mu_g(x) = 0$.
  - $H_A$: $\mu_M - \mu_g(x) > 0$.
  - Using a 1% probability of Type I error ($\alpha = 0.01$).
  - $\alpha = 5.179741 \times 10^{-43}$ shows that the null hypothesis can be rejected.

Chirp Pattern Cluster

Training portion of time series

Pattern cluster applied to test time series
Time Series View of Chirp

Pattern, p, is from training process

Noisy Sinusoidal Pattern Cluster

Pattern cluster applied to test time series
Time Series View of Noisy Sinusoidal

Pattern, \( p \), is from training process

Noise Pattern Cluster

Training portion of time series

Pattern cluster applied to test time series
**Welding Application**

- **Welding Process**
  - Two pieces of metal joined into one by making a joint between them
  - Arcing current is created between welder and metal to be joined
  - Wire is pushed out of welder
  - Tip of wire melts, forming a droplet of metal that elongates (sticks out) until it releases
  - Goal: Predict when droplet will release
    - Can’t be done by traditional methods

**Welding Data Set**

- **Goal: Predict when a droplet will release**
- **Four time series**
  - Release (event), 1kHz sampling rate (~5000 data points)
  - Stickout, 1kHz sampling rate (~5000 data points)
  - Current, 5kHz sampling rate (~35,000 data points)
  - Voltage, 5kHz sampling rate (~35,000 data points)
  - Data not originally synchronized
- **First Pass**
  - Used release time series as event function
  - Used stickout as time series
    - “not too reliable”
  - Used a set of phase spaces \( Q \) from dimension 1 to 16
  - Generated 16 temporal patterns
Welding Initial Results

- **Training Set**
  - Runs $\alpha = 0$
  - Means $\alpha = 3.02 \times 10^{-49}$
  - true positives: 125 (5.6%), false positives: 99 (4.4%)
  - true negatives: 2005 (89.3%), false negatives: 17 (0.8%)
  - 94.8% accuracy

- **Testing Set**
  - Runs $\alpha = 0$
  - Means $\alpha = 1.34 \times 10^{-51}$
  - true positives: 97 (3.5%), false positives: 52 (1.9%)
  - true negatives: 2470 (89.1%), false negatives: 55 (2.0%)
  - 95.1% accuracy
Statistical Tests

- **Runs Test**
  - H₀: There is no difference between the matched time series and the whole time series.
  - H₁: There is significant difference between the matched time series and the whole time series.
  - Using a 1% probability of Type I error ($\alpha = 0.01$).

- **Difference of two independent means**
  - Although the two populations are probably dependent, this can be ignored because it makes the statistics more conservative, i.e., it will tend to overestimate the Type I error.
  - H₀: $\mu_M - \mu_{g(X)} = 0$.
  - H₁: $\mu_M - \mu_{g(X)} > 0$.
  - Using a 1% probability of Type I error ($\alpha = 0.01$).

ICN Time Series

- **ICN is a pharmaceutical company whose stock trades on the NYSE**
- **Training Time Series**
  - Daily open price for 1st 126 trading days of 1990
- **Testing Time Series**
  - Daily open price for the 2nd 127 trading days of 1990
- **Stock prices tend to grow exponentially**
  - A filter is applied to the time series
    \[
    Z = \left\{ z_i = \frac{(1 - B)}{B} x_i \right\}
    \]
ICN Train Phase Space

Filter

ICN Test Phase Space

Filter

Pattern, p, is from training process
ICN Initial Results

- **Training Set**
  - Using Buy and Hold Strategy: -26.2% (125 days)
  - Using Temporal Patterns: 51.0% (8 days)
  - Runs $\alpha = 2.21 \times 10^{-3}$
  - Means $\alpha = 2.42 \times 10^{-2}$

- **Testing Set**
  - Using Buy and Hold Strategy: -24.1% (126 days)
  - Using Temporal Patterns: 17.3% (22 days)
  - Runs $\alpha = 3.63 \times 10^{-3}$
  - Means $\alpha = 2.54 \times 10^{-4}$
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- References
THANK YOU!

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