MATH 83, Submit Homework #8
Due in class on Friday, April 4th

All numerical answers must show and be correct to 4 decimal places. Easy to follow work must be attached (stapled). Except for #1, no partial credit will be given.

Consider the initial value problem: \( y' - 2ty + 1, y(0) = 2 \).

1. (3 pts) Solve this initial value problem mathematically.
2. (3 pts) Solve this initial value problem to find a numerical approximation to \( y(.8) \) using the Euler, Improved Euler, and Runge-Kutta methods. Use a step size of \( h = 0.2 \). Fill in the appropriate spaces in the chart.
3. (1 pt) Calculate the exact solution of \( y(.8) \). Fill in the appropriate space in the chart.

\[
y(t) = e^{t^2} \left[ \int_0^t e^{-s^2} ds + 2 \right]
\]

<table>
<thead>
<tr>
<th>Euler Approximation to ( y(.8) )</th>
<th>Improved Euler Approximation to ( y(.8) )</th>
<th>Runge-Kutta Approximation to ( y(.8) )</th>
<th>Actual solution value of ( y(.8) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1533</td>
<td>5.0171</td>
<td>5.0401</td>
<td>5.0402</td>
</tr>
</tbody>
</table>
1. \( y' - 2ty = 1 \)
   \[
   \mu(t) = e^{2s} = e^{-t^2}
   \]
   
   \( \frac{d}{dt} (y(s)e^{-s^2}) = e^{-t^2} \)
   
   \[
   \int_0^t (y(s)e^{-s^2}) ds = \int_0^t e^{-s^2} ds
   \]
   
   \[
   y(s)e^{-s^2} \bigg|_0^t = \int_0^t e^{-s^2} ds \Rightarrow y(t)e^{-t^2} - y(0) = \int_0^t e^{-s^2} ds
   \]
   
   \[
   y(t)e^{-t^2} = \int_0^t e^{-s^2} ds + 2 \Rightarrow y(t) = e^{t^2} \left[ \int_0^t e^{-s^2} ds + 2 \right]
   \]

The exact solution using Maple

```maple
> A := evalf(int(exp(-t^2), t = 0 . . . 0.8));
A := .6576698563
> exactsoln := exp((.8)^2)*(A+2);

exactsoln := 5.040220065
```

*Note: The TI-89 returns a value of \( .657667 \) when 
\( \text{nInt}(e^{(-x^2)}), x, 0, .8) \) is executed. 
The TI-89 has the goal of 6 significant digits. 
On the TI-89, \( e^{(.8^2)} \times \left( \text{nInt}(e^{(-x^2)}), x, 0, .8) + 2 \right) \)
\( = 5.04022 \)
### Euler

<table>
<thead>
<tr>
<th>tn</th>
<th>yn</th>
<th>y(n+1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.0000</td>
<td>2.2</td>
</tr>
<tr>
<td>0.2</td>
<td>2.2000</td>
<td>2.576</td>
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<tr>
<td>0.4</td>
<td>2.5760</td>
<td>3.18816</td>
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<tr>
<td>0.6</td>
<td>3.1882</td>
<td>4.15332</td>
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<tr>
<td>0.8</td>
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</tbody>
</table>

### Improved Euler

<table>
<thead>
<tr>
<th>tn</th>
<th>yn</th>
<th>2tnyn+1</th>
<th>A</th>
<th>B = y(n+1)</th>
<th>(\frac{2}{3}(A+C))</th>
<th>y(n+1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.0000</td>
<td>1.0000</td>
<td>2.2000</td>
<td>0.2</td>
<td>1.88</td>
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<td>0.2</td>
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### Runge Kutta

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<th>tn</th>
<th>yn</th>
<th>kn1</th>
<th>kn2</th>
<th>kn3</th>
<th>kn4</th>
<th>y(n+1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>1.0000</td>
<td>1.42000</td>
<td>1.42840</td>
<td>1.91427</td>
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<td>1.91481</td>
<td>2.48711</td>
<td>2.52145</td>
<td>3.23306</td>
<td>2.79254</td>
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<tr>
<td>0.4</td>
<td>2.7925</td>
<td>3.23403</td>
<td>4.11594</td>
<td>4.20413</td>
<td>5.36003</td>
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<tr>
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<td>5.36041</td>
<td>6.83760</td>
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<td>5.04009</td>
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<td>5.0401</td>
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<td></td>
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