In each of Problems 1 through 6 use Euler's formula to write the given expression in the form $a + ib$.

1. $\exp(1 + 2i)$
2. $\exp(2 - 3i) = e^2 e^{-3i} = e^2 (\cos 3 - i \sin 3)$.
3. $e^{i\pi} = \cos \pi + i \sin \pi = -1$.
4. $\exp\left(2 - \frac{\pi}{2}i\right) = e^2 (\cos \frac{\pi}{2} - i \sin \frac{\pi}{2}) = -e^2 i$.

In each of Problems 7 through 16 find the general solution of the given differential equation.

7. $y'' - 2y' + 2y = 0$
8. $y'' - 2y' + 6y = 0$
9. $y'' + 2y' - 8y = 0$
10. $y'' + 2y' + 2y = 0$

The characteristic equation is $r^2 + 2r + 2 = 0$, so roots are $r = -1 \pm \sqrt{2}$.

So roots are complex conjugates (with $\lambda = -1$ and $\mu = 1$).

Thus $f(t) = e^{-t} \left( c_1 \cos t + c_2 \sin t \right)$ and the general solution is $y(t) = c_1 e^{-t} \cos t + c_2 e^{-t} \sin t$.

In each of Problems 17 through 22 find the solution of the given initial value problem. Sketch the graph of the solution and describe its behavior for increasing $t$.

17. $y'' + 4y = 0$, $y(0) = 0$, $y'(0) = 1$

17. The characteristic equation is $r^2 + 4 = 0$, with roots $r = \pm 2i$. Hence the general solution is $y = c_1 \cos 2t + c_2 \sin 2t$. Its derivative is $y' = -2c_1 \sin 2t + 2c_2 \cos 2t$.

Based on the first condition, $y(0) = 0$, we require that $c_1 = 0$. In order to satisfy the condition $y'(0) = 1$, we find that $2c_2 = 1$. The constants are $c_1 = 0$ and $c_2 = 1/2$.

Hence the specific solution is $y(t) = \frac{1}{2} \sin 2t$.

As $t \to \infty$ oscillates between $\frac{1}{2}$ and $-\frac{1}{2}$ (in the steady state).