REVIEW GUIDE FOR FINAL EXAM

[Note: This does not cover everything that is on the final (There are 16 problems on the final.). In addition to going over these problems you should look over your Review Guides for Exams 1 – 3 and over Exams 1-3.]

1. (Example 6 from Section 1.2 lecture). A drug is absorbed by the body at a rate proportional to the amount \( y(t) \) present in the bloodstream. Suppose that initially there is no drug in the bloodstream but at time \( t = 0 \) the patient begins to receive the drug intravenously at the constant rate of 15 milligrams per hour. The drug is absorbed at the rate of \( 0.5y(t) \) per hour.
   a) Set up an I.V.P. (differential equation plus initial condition) to model this situation.
   b) Sketch a phase line for the model and classify the equilibrium solution.
   c) Express the long-term behavior of the solution to the I.V.P. of part a) as a limit.

2. The acceleration of a Lamborghini is proportional to the difference between 250 km/h and the velocity of the car. Set up a differential equation to model the motion of the car.

3. Given \( y'' = y^2 t^3 \)
   a) Find an explicit one-parameter family of solutions.
   b) Find an explicit solution for the initial value problem, \( y'' = y^2 t^3 \), \( y(2) = 0 \).

4. Find the specific solution of \( ty' = \sin t \), \( y(1) = 2 \)

5. Find the general solution of \( y'' - 2y' = t - 5 \).

6. Find a specific solution of \( y'' + 4y = 5 \), \( y(0) = 0 \), \( y'(0) = 1 \).

7. Solve \( y' - y = e^{2t} \) for an explicit one-parameter family of solutions.

8. Given \( u(t) = e^{-6t}(-2\cos 3t + 2\sin 3t) \)
   a) Convert into the amplitude phase-angle form.
   b) Construct the original differential equation for which \( u(t) = e^{-6t}(-2\cos 3t + 2\sin 3t) \) is a solution.
   c) Describe the type of motion modeled by the solution \( u(t) = e^{-6t}(-2\cos 3t + 2\sin 3t) \).

9. Suppose a 1-lb weight stretches a spring 6 inches beyond its natural length. An external force equal to \( \frac{1}{4}\cos 8t \) is acting on the spring. If the weight is started in motion from its equilibrium position with an upward velocity of 4 ft/sec and there is no damping, set up an initial value problem whose solution will give an equation of the motion of the mass + spring. Describe the type of motion this model describes.

10. Find the specific solution of \( x' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} x \), \( x(0) = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \). Express it three different ways.

11. Find the Laplace transforms or inverse transforms of the following:
   a) \( L(e^{2t} \cos t) \)
   b) \( L^{-1}\left(\frac{1}{s(s-2)^2}\right) \)
   c) \( L^{-1}\left(\frac{s}{s^2 + 2s + 17}\right) \)
12. Consider the two tanks shown. Tank A initially contains 10 lbs.
of salt dissolved in 50 gallons of water. Tank B initially contains 50
gallons of pure water. Pure water flows into tank A and brine flows
between tanks A and B and out of B as shown. Let \( x_1(t) \) and \( x_2(t) \) denote
the amounts of salt in tanks A and B, respectively, at any time \( t \).

a) State the equation for \( \frac{dx_1}{dt} \) (rate of change of salt in tank A for the
is the initial vector?
c) For how long is a model of the system valid if tank A has a capacity of 100 gallons?

Two Supplementary Problems:
1. Solve \( y' + 2ty = 1 \), \( y(0) = 1 \)
2. Find the general solution of \( \mathbf{x}' = \begin{pmatrix} 4 & 2 \\ 3 & -1 \end{pmatrix} \mathbf{x} \). Express the general solution in three ways: 1) As a
linear combination of solutions, 2) in terms of the fundamental matrix, \( \Psi(t) \), 3) as a system of two
equations.