Section 7.9, Nonhomogeneous Linear Systems (Variation of Parameters)
(Note: The method below is not as in the text, but is based on it.)

I. To find the general solution for the system \( x' = Ax + g(t) \):
   1. First find the general solution \( x_c(t) \) of \( x' = Ax \).
   2. Use Variation of Parameters to find a particular solution \( X(t) \) of \( x' = Ax + g(t) \).
   3. The general solution of \( x' = Ax + g(t) \) is \( x(t) = x_c(t) + X(t) \).

II. The method of Variation of Parameters for systems:

   Suppose the homogeneous equation \( x' = Ax \) has the general solution
   \[
   x_c(t) = c_1 x^{(1)}(t) + \cdots + c_n x^{(n)}(t). 
   \]

   Then \( X(t) \) will have the form \( X(t) = u_1(t) x^{(1)}(t) + \cdots + u_n(t) x^{(n)}(t) \) obtained by allowing the parameters to "vary".

   To find the values of the \( u_i'(t) \), solve the system
   \[
   u_1'(t) x^{(1)}(t) + \cdots + u_n'(t) x^{(n)}(t) = g(t) 
   \]
   (or equivalently, \( \Psi(t) u'(t) = g(t) \)).

   This system can be solved using Cramer’s rule or by reducing an augmented matrix.

   Integrate each \( u_i'(t) \) to find \( c_i(t) \).

   Example: Solve \( x'(t) = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ e^t \end{bmatrix} \) if \( \psi(t) = \begin{bmatrix} e^{-t} & 3e^{4t} \\ -e^{-t} & 2e^{4t} \end{bmatrix} \).