Sections 3.7, The Method of Variation of Parameters

Steps in applying the method of Variation of Parameters to find $Y(t)$ for

$$P(t)y'' + Q(t)y' + R(t)y = G(t) \quad \text{(N)}$$

1. Find the complementary solution $y_c(t) = c_1y_1(t) + c_2y_2(t)$ of the corresponding homogeneous equation

$$P(t)y'' + Q(t)y' + R(t)y = 0 \quad \text{(H)}$$

2. The form of $Y(t)$ is obtained by varying the parameters of $y_c(t)$.

$$Y(t) = u_1(t)y_1(t) + u_2(t)y_2(t)$$

$$u_1'(t)y_1(t) + u_2'(t)y_2(t) = 0$$

3. Solve the system

$$u_1'(t)y_1'(t) + u_2'(t)y_2'(t) = \frac{G(t)}{P(t)} \text{ for } u_1'(t) \text{ and } u_2'(t).$$

Note that the coefficient matrix of this system is the Wronskian of the solutions of (H)

4. Integrate $u_1'(t)$ and $u_2'(t)$ to find $u_1(t)$ and $u_2(t)$.

5. Substitute the values of $u_1(t)$ and $u_2(t)$ from step 4 into $Y(t) = u_1(t)y_1(t) + u_2(t)y_2(t)$.

Example 1. Find the general solution of $y'' - 4y' = 6e^t$. 
Example 2. Find the general solution of \((t^2 - 1)y'' - 2ty' + 2y = (t^2 - 1)^2, t > 1\), if
\[ y_c(t) = c_1t + c_2(t^2 + 1). \]