Q1: Use the graph of \( y = f(x) \) and the two points A and B, to answer the following.

\[ y = f(x) \]

\[ A \left( x, f(x) \right) \quad B \left( x+h, f(x+h) \right) \]

Fill in the blanks:

A. The \((x, y)\) coordinates of the points A and B are: A\((x, f(x))\) & B\((x+h, f(x+h))\)

The formula for the slope of the line connecting A and B is

\[ \frac{f(x+h) - f(x)}{h} \]

B. Define the derivative function \( f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \)

Q2: The graph of \( f(x) = \sqrt{x} \) is shown. Answer A-D.

A. Find \( f'(x) \)

\[ f(x) = x^{\frac{1}{2}} \quad f'(x) = \frac{1}{2} x^{-\frac{1}{2}} \]

B. Find the equation of the tangent line to \( f(x) \) at \( x = 4 \)

AND sketch the tangent line on the graph.

\[ f(4) = 2 \quad \Rightarrow \quad (4, 2) \text{ point} \]

\[ f'(4) = \frac{1}{2} \cdot \frac{1}{\sqrt{4}} = \frac{1}{4} \quad \Rightarrow \quad \text{slope} = \frac{1}{4} \]

\[ y - 2 = \frac{1}{4} (x - 4) \]

\[ y = \frac{1}{4} x - 1 + 2 \]

\[ y = \frac{1}{4} x + 1 \]

C. Find a linear approximation of \( \sqrt{4.8} \).

\[ f(4.8) \approx f(4) + f'(4)(0.8) \]

\[ \approx 2 + \frac{1}{4} (0.8) \]

\[ \approx 2.2 \]

Use the tangent line to approximate

\[ y = \frac{1}{4} (4.8) + 1 \]

\[ = 1.2 + 1 \]

\[ = 2.2 \]

\[ \Rightarrow \sqrt{4.8} \approx 2.2 \]
Q5: If a cup of hot coffee is left on a counter top. It's temperature in °F t minutes after it was left on the counter is given by

\[ T(t) = 74 + 103e^{-0.025t} \]

A. What was the temperature of the coffee when it was left on the counter? \(177°F\ (74+103)\)

B. What is the significance of the number 74?

It is the temperature of the room.

C. Sketch the graph of the function, indicating on the y-axis all important values (those found in A & B).

D. Find \( T'(t) \)

\[ T'(t) = 0 + 103e^{-0.025t}(-0.025) \]

\[ T'(t) = -2.575e^{-0.025} \]

E. Write a statement that interprets the values \( T(10) \) and \( T'(10) \).

\[ T(10) = 154.216 \quad T'(10) = -2.005 \]

After 10 minutes the temp. of the coffee is 154.216 °F.

And it is decreasing at a rate of 2.005 °F/minute.

Q6: Find the derivatives:

A. \( y = \frac{3}{x^2} + \frac{x^2}{3} \)

\[ y = 3x^{-2} + \frac{1}{3}x^2 \]

\[ y' = -6x^{-3} + \frac{2}{3}x \]

or \( y' = \frac{-6}{x^3} + \frac{2}{3}x \)

B. \( y = 5 \cdot \ln \left( \frac{t}{2} \right) \)

\[ y' = 5 \cdot \frac{1}{t} \left( \frac{1}{2} \right) \]

\[ y'' = \frac{5}{t} \]
Q7: Given the first derivative of \( f(t) \) is \( f'(t) = -4te^{-0.2t} + 20e^{-0.2t} \).

A. Find \( f''(t) \).
\[ f''(t) = \left[ 4e^{-0.2t} + (-4t)e^{-0.2t}(-0.2) \right] + 20e^{-0.2t}(-0.2) \]
\[ = -4e^{-0.2t} + 8te^{-0.2t} - 4e^{-0.2t} \]
\[ f''(t) = -8e^{-0.2t} + 8te^{-0.2t} \]

B. Find the one critical point of \( f(t) \). Determine the behavior of \( f(t) \) at this point.
\[ 0 = f'(t) \]
\[ 0 = -4te^{-0.2t} + 20e^{-0.2t} \]
\[ 0 = e^{-0.2t}(-4t + 20) \]
\[ 0 = -4t + 20 \]
\[ t = 5 \]
\[ f'(5) = -4e^{-0.2(5)} < 0 \]
\[ \therefore \text{ } f \text{ is } \nearrow \text{ at } 5 \]
Answer: Local max at \( t = 5 \).

C. Where does \( f(t) \) have a point of inflection?
\[ f''(t) = 0 \]
\[ 0 = e^{-0.2t}(-8 + 8t) \]
\[ 0 = -8 + 8t \]
\[ 8 = 8t \]
\[ t = 10 \]
Point of inflection at \( t = 10 \).

Q8: Find the derivatives and simplify.
\[ f(x) = \frac{\cos(4x)}{x^2} \]
\[ f'(x) = \frac{x^2(-\sin(4x) \cdot 4) - 2x \cos(4x)}{x^4} \]
\[ = \frac{-4x \sin(4x) - 2 \cos(4x)}{x^3} \]
\[ = -4x \frac{\sin(4x)}{x^3} - 2 \frac{\cos(4x)}{x^3} \text{ or } -2 \frac{2x \sin(4x) + \cos(4x)}{x^3} \]
Q3: During a flood, the water level in a river first rose faster and faster, then rose more and more slowly until it reached its highest point, then went back down to its pre-flood level at a constant rate. Sketch a graph of water depth as a function of time. Mark any points of inflection with an 'I' and any critical points with a 'C'.

![](graph1.png)

Q4: To the right is the global graph of \( y = f'(x) \).
Use the graph to answer the questions about \( f(x) \).

A. What are the critical points of \( f(x) \)?
\[
\chi = -3, -1, 1
\]

B. At what \( x \) - values does \( f(x) \) have a local minimum?
\[
\chi = 2
\]

C. At \( x = 1.5 \) is \( f(x) \) increasing or decreasing? \( \text{decreasing (since } f' < 0 \text{)} \)
Concave up or concave down? \( \text{concave up (since } f'' > 0 \text{)} \)

D. Approximately where does \( f(x) \) have points of inflection?
\[
\chi = -3, -1.7, 1 \text{ (where } f' \text{ changes from } + \text{ to } - \text{ or } - \text{ to } + \text{)}
\]

E. Circle T (true) or F (false) for the statements about the values of \( f(x) \):
1. T  F  \( f(-3) < f(-1) \)
2. T  F  \( \text{If } f(-2) = 5, \text{ then } f(-1) > 5 \)
3. T  F  \( f(2) > f(1) \)
4. T  F  \( f(x) \) has a global minimum at \( x = 2 \)