Show all work. Label all answers to word problems.

Show 3 significant digits to the right of the decimal point in calculator answers

**Q1:** The average weight (in pounds) of American men in their sixties as a function of their heights (in inches) is given in the table.

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<tr>
<th>height (h)</th>
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<th>69</th>
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<tbody>
<tr>
<td>weight (w)</td>
<td>166</td>
<td>171</td>
<td>176</td>
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<td>186</td>
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</tr>
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</table>

\[ \Delta w = 5, 5, 5, 5, 5 \]

A. How do you know from the table that weight \( w \) is a linear function of height \( h \)?

\[ \Delta w \text{ is a constant, } 5, \text{ for every change } 1 \text{ in } h. \]

B. What is the slope of the line, give units and explain what it means.

\[ \frac{\Delta w}{\Delta h} = \frac{5}{1} \text{ lbs/\text{ inch}} \]

C. Find the formula that expresses the weight as a function of height.

Using point \((68, 166)\) and slope 5

\[ y - y_0 = m(x - x_0) \]

\[ y - 166 = 5(x - 68) \]

\[ y = 5x - 340 + 166 \]

\[ y = 5x - 174 \]

**Q2:** On the axes, sketch a smooth curve (i.e. no sharp corners, no breaks) which passes through the point \( P(3, 4) \), and which clearly satisfies the following conditions.

- Concave up to the left of \( P \)
- Concave down to the right of \( P \)
- Increasing for \( x > 0 \)
- Decreasing for \( x < 0 \)
Q3:

Show your work in the space below.  

Place your answers, rounded to two decimal places in the boxes after each question.

Lisinopril is an ACE inhibitor derived from the venom of a Brazilian pit viper frequently used in the treatment of hypertension. Because of Lisinopril's relatively long half life of 12 hours, patients need to take a dose just once per day. A patient takes his first dose, 20 mg, at 6 pm on Saturday.

A. How many milligrams remain in the patient's body right before he takes his next 20 mg at 6 pm on Sunday?

Ans: 5 gms

B. After the first dose on Saturday, how many hours does it take for the amount of Lisinopril in the patient's body to decrease to 7 mg?

Ans: 18.175 hours

Note: H can be answered without the exact equation since the half life is 12 hours, after 24 hours 1/4 remains.

\[
\begin{align*}
\text{t} = 0, & 20 \text{ gms} \\
\text{t} = 12, & 10 \text{ gms} \\
\text{t} = 24, & 5 \text{ gms}
\end{align*}
\]

B. \( Q = Q_0 e^{kt} \)

\[
\frac{1}{2} Q_0 = Q_0 e^{-12k} \\
\frac{1}{2} = e^{-12k} \\
\ln \frac{1}{2} = 12k \\
k = \frac{\ln \frac{1}{2}}{12} \\
k = -0.0578
\]

\[
\begin{align*}
Q = 20 e^{-0.0578 t} \\
7 = 20 e^{-0.0578 t} \\
\ln \frac{7}{20} = \ln (\frac{3}{12}) \\
t = \frac{\ln \frac{7}{20}}{-0.0578} \\
t = 18.163 \text{ hours}
\end{align*}
\]

Q4: In the mathematical folk tale "One Grain of Rice", a girl receives a reward that starts with one grain of rice on day one, two grains of rice on day two, four on day three and eight on day four. Each day, she receives double the number from the day before. How many grains of rice does she receive on the 25th day?

A. (Circle) How many grains of rice does she receive on the 25th day?

i. 50  
ii. 625  
iii. 1,024  
iv. 16,777,216  
v. 33,554,432

B. What formula did you use to calculate your answer? \( y = 2^{t-1} \), \( y = \frac{2^t}{2} \)
Q7: If \( t \) is years since 2000, the population, in thousands, of a small metropolitan area is given by

\[ P = 50e^{0.0351t} \]

A. What is the continuous percentage rate of change?

\[ 0.0351 = 3.51\% \]

B. What is the equivalent annual percentage rate of change?

\[ P = 50a^t \quad \text{where} \quad a = e^k \]

\[ \frac{\log a}{\log e} = r = a - 1 \]

\[ r = 0.0351 \]

\[ r - 1 \]

\[ 3.57\% \]

C. Estimate the instantaneous rate of change in the year 2003. (label)

\[ \text{derivative} \]

Find average rate of change at \( t = 3 \), by choosing a point close to 3, such as 2.99, 2.999 or 3.01 or 3.001 and calculating

\[ \frac{dy}{dx} = \frac{f(3 + h) - f(3)}{h} \]

\[ f(3.001) = 50e^{0.0351(3.001)} = 55.55414359 \]

\[ f(3) = 55.5529368 \]

\[ \frac{f(3.001) - f(3)}{0.001} = 1.94991 \]

1.95 thousand people/year

\( f'(3) \times \frac{1,950 \text{ people}}{\text{year}} \)
Q5: Below are two graphs of polynomials with dominating term $ax^n$. For each state the sign of $a$ and the minimum possible value of $n$.

A.  
1. sign of $a$: $+$  
2. minimum possible $n$: $4$  

Since 3 turns $n$ must be at least 4, could be 6, 8, 10, ...

B.  
1. sign of $a$: $-$  
2. minimum possible $n$: $3$  

$n$ must be odd & at least 3. Could be 5, 7, 9, ...

Q6: Temperatures in Town A oscillate daily between $30^\circ$ F at 4 am and $60^\circ$F at 4 pm. Write the following formulas: (Note: you do not need to show the graphs, the grid is here in case you need it)

A. Temperature, $T_A$, in Town A, in terms of time, $t$, where time is measured in hours from 4 am.

$$T_A = -15 \cos \left( \frac{\pi}{12} t \right) + 45$$

Amplitude = $60 - 30 = 15$

Period = $24 \Rightarrow \frac{2\pi}{12} = \frac{\pi}{6}$

Start at 4 am = $30^\circ$

B. Temperature in Town B, where the temperatures are consistently $10^\circ$F colder than in Town A and measured from 4 am.

$$T_B = -15 \cos \left( \frac{\pi}{12} t \right) + 35$$

Shift down by $10^\circ$

$$(-15 \cos \left( \frac{\pi}{12} t \right) + 45) - 10$$

C. Temperature in Town A, in terms of time where time is measured in hours from midnight.

$$T = -15 \cos \left( \frac{\pi}{12} (t-4) \right) + 45$$

or $T = -15 \cos \left( \frac{\pi}{12} \left( t - \frac{4}{12} \right) \right) + 45$

Start at midnight $\Rightarrow$ Shift horizontally 4 units (difference between 12 am & 4 am) to the right $\Rightarrow$

$t - 4$
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