Decimal answers should be rounded to three places to the right of the decimal point.
Word problems should include units in your answers.

Q1: The price-demand and total cost functions for the production and sale of $x$ sweatshirts are

$$p = 36 - 0.03x \quad \quad C(x) = 6x + 750$$

A. What is the total revenue function?

$$R(x) = x \cdot p = 36x - 0.03x^2$$

B. What is the total profit function?

$$P(x) = p(x) - C(x) = 36x - 0.03x^2 - (6x + 750) = 30x - 0.03x^2 - 750$$

C. Use only the values below and marginal analysis to find the answers to 1–3

$$P(250) = 4,875 \quad P'(250) = 15 \quad P(250) = 19.50 \quad P'(250) = -0.018$$

1. Approximate the profit from the sale of the 251st sweatshirt. Answer: $15$

2. Approximate the average profit from the sale of 251 sweatshirts. Answer: $19.482$

$$19.50 + (-0.018) = 19.482$$

3. Approximate the total profit from the sale of 255 sweatshirts. Answer: $4,950$

$$P(255) \approx P(250) + P'(250) \cdot 5 = 4,875 + 15 \cdot 5 = 4,950$$

Q2: The function $Z(t)$ models the ozone level $Z$, measured in parts per billion (ppb), on a hot summer day, where $t$ is time in hours and $t = 0$ corresponds to 9 am. Write a single statement that interprets these values:

$$Z(3) = 107 \quad Z'(3) = 6$$

At noon the ozone level is 107 ppb and is increasing at a rate of $6 \frac{ppb}{hr}$.
Q3: A cesium isotope has a half-life of 30 years. What is the continuous compound rate of decay?

\[ Q = Q_0 e^{rt} \]

\[ \frac{1}{2} Q_0 = Q_0 e^{30r} \]

\[ \frac{1}{2} = e^{30r} \]

\[ \ln \left( \frac{1}{2} \right) = 30r \]

\[ r = \frac{\ln \left( \frac{1}{2} \right)}{30} = -0.0231 \]

B. Circle all true statements about the function \( f(t) = e^{-t} \)

1. The domain of \( f(t) \) is all Real numbers.
2. \( f'(t) < 0 \) on the entire domain of \( f(t) \).
3. \( f''(t) < 0 \) on the entire domain of \( f(t) \).
4. \( \lim_{t \to \infty} f(t) = 0 \)

Q4: Related Rates: A block of ice, in the shape of a cube, is hung from a hook in a room. If the ice is melting uniformly, so that the shape of the block is always a cube, both the length of a side of the cube and the volume of the cube are changing.

If the length of a side is decreasing at a rate of 2 inches per hour, at what rate is the volume of the cube changing when each side is 10 inches long?

A. What is the equation that relates the volume, \( V \), and \( s \), the length of each side?

Equation: \( V = s^3 \)

(If you don't know, I will sell it to you for two points).

B. Rewrite the statement of the problem using correct notation:

Given \( \frac{ds}{dt} = -2 \) \( \text{in/hr} \) find \( \frac{dV}{dt} \) when \( s = 10 \)

C. Solve.

\[ \frac{dV}{dt} = 3 s^2 \cdot \frac{ds}{dt} \]

\[ \frac{dV}{dt} = 3(10)^2 (-2) = -600 \text{ in}^3/\text{hr} \]
Q5: Given the information about $f(x)$, $f'(x)$ and $f''(x)$ in the three charts below, sketch a graph of the function $f(x)$, which is continuous on $(-\infty, \infty)$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-4</th>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>-2</td>
<td>3</td>
<td>-1</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

$f'(x)$: $++ 0 -- \text{ND} + + 0 + +$

$f(x)$:

-2 0 2

$f''(x)$: $-- -- \text{ND} -- 0 + +$

$f(x)$:

0 2

Graph of $f(x)$ showing critical points and intervals of increase and decrease.
Q6: Answer the questions about the unknown function \( f(x) \) whose first and second derivatives are:

\[
\begin{align*}
    f'(x) &= 4x^3 - 12x^2 \\
    &= 4x^2(x - 3) \\
    &= 0, \quad x = 3
\end{align*}
\]

\[
\begin{align*}
    f''(x) &= 12x^2 - 24x \\
    &= 12x(x - 2) \\
    &= 0, \quad x = 2
\end{align*}
\]

A. Compete a sign chart for \( f'(x) = 4x^3 - 12x^2 \)

B. Compete a sign chart for \( f''(x) = 12x^2 - 24x \)

C. Using your information in A & B, answer the following. Your answers must be consistent with A & B.

1. At what value(s) of \( x \) does \( f(x) \) have a local maximum? \[ \underline{\text{no local maximum}} \]
2. On what interval(s) is \( f(x) \) increasing? \[ (3, +\infty) \]
3. At what value(s) of \( x \) does \( f(x) \) have a point of inflection? \[ 0, 2 \]
4. On what interval(s) is \( f(x) \) concave up? \[ (-\infty, 0) \cup (2, +\infty) \]
Decimal answers should be rounded to three places to the right of the decimal point. Word problems should include units in your answers.

Q1: The price-demand and total cost functions for the production and sale of \( x \) sweatshirts are
\[
p = 36 - 0.03x \quad C(x) = 6x + 750
\]

A. What is the total revenue function?
\[
R(x) = x \cdot p = 36x - 0.03x^2
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B. What is the total profit function?
\[
P(x) = R(x) - C(x) = 36x - 0.03x^2 - (6x + 750) = 30x - 0.03x^2 - 750
\]

C. Use only the values below and marginal analysis to find the answers to 1 - 3

\[
P(300) = 5,550 \quad P'(300) = 12 \quad \bar{P}(300) = 18.50 \quad \bar{P}'(300) = -.022
\]

1. Approximate the profit from the sale of the 301st sweatshirt. Answer: \( \$12 \)

2. Approximate the average profit from the sale of 301 sweatshirts. Answer: \( \$18.478 \)

\[
\bar{P}(301) = \bar{P}(300) + \bar{P}'(300)
\]

\[
= 18.50 - .022
\]

3. Approximate the total profit from the sale of 305 sweatshirts. Answer: \( \$5,610 \)

\[
P(305) \approx \bar{P}(300) + P'(300) \cdot 5
\]

\[
= 5,550 + 12.5 = 5,610
\]

Q2: The function \( Z(t) \) models the ozone level \( Z \), measured in parts per billion (ppb), on a hot summer day, where \( t \) is time in hours and \( t = 0 \) corresponds to 9 am. Write a single statement that interprets these values:

\[
Z(4) = 112 \quad Z'(3) = 4
\]

At 1 pm the ozone level is 112 ppb and is increasing at a rate of 4 ppb per hour.
Q3: A cesium isotope has a half-life of 25 years. What is the continuous compound rate of decay?

\[ Q = Q_0 e^{rt} \]

\[ \frac{1}{2} Q_0 = Q_0 e^{25r} \]

\[ \frac{1}{2} = e^{25r} \]

\[ \ln \frac{1}{2} = 25r \]

\[ r = \frac{\ln \frac{1}{2}}{25} \]

\[ r = -0.0277 \]

B. Circle all true statements about the function \( f(t) = e^{-t} \)

1. The domain of \( f(t) \) is all Real numbers.
2. \( f'(t) > 0 \) on the entire domain of \( f(t) \).
3. \( f''(t) > 0 \) on the entire domain of \( f(t) \).
4. \( \lim_{t \to -\infty} f(t) = \infty \)

Q4: Related Rates: A block of ice, in the shape of a cube, is hung from a hook in a room. If the ice is melting uniformly, so that the shape of the block is always a cube, both the length of a side of the cube and the volume of the cube are changing.

If the length of a side is decreasing at a rate of 2 inches per hour, at what rate is the volume of the cube changing when each side is 10 inches long?

A. What is the equation that relates the volume, \( V \), and \( s \), the length of each side?

Equation: \( V = s^3 \)

(If you don't know, I will sell it to you for two points).

B. Rewrite the statement of the problem using correct notation:

Given \( \frac{ds}{dt} = -2 \) in/hr find \( \frac{dV}{dt} \) when \( s = 10 \)

C. Solve.

\[ \frac{dV}{dt} = 3s^2 \cdot \frac{ds}{dt} \]

\[ \frac{dV}{dt} = -600 \text{ in}^3/\text{hr} \]

\[ \frac{dV}{dt} = 3(10)^2 (-2) \]

\[ \frac{dV}{dt} = -600 \]
Q5: Given the information about \( f(x), f'(x) \) and \( f''(x) \) in the three charts below, sketch a graph of the function \( f(x) \), which is continuous on \((-\infty, \infty)\).

\[
\begin{array}{c|c|c|c|c|c}
 x & -4 & -2 & 0 & 2 & 4 \\
 \hline
 f(x) & 4 & 2 & -1 & 3 & -1 \\
\end{array}
\]

\[
f'(x) - - 0 -- \text{ND} + 0 - - \\
f(x) -2 \circ 0 \circ 2
\]

\[
f''(x) + 0 -- \text{ND} - - - \\
f(x) -2 0
\]
Q6: Answer the questions about the unknown function $f(x)$ whose first and second derivatives are:

$$f'(x) = 4x^3 - 12x^2$$

$$f''(x) = 12x^2 - 24x$$

A. Compete a sign chart for $f'(x) = 4x^3 - 12x^2$

<table>
<thead>
<tr>
<th>$f'$</th>
<th>$-$</th>
<th>$-$</th>
<th>$+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>$-$</td>
<td>$0$</td>
<td>$3$</td>
</tr>
</tbody>
</table>

$$f''(x) = 12x(x-2)$$

$$f'' = 0 \implies x = 0, x = 2$$

B. Compete a sign chart for $f''(x) = 12x^2 - 24x$

<table>
<thead>
<tr>
<th>$f''$</th>
<th>$-$</th>
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<th>$+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f'$</td>
<td>$-$</td>
<td>$0$</td>
<td>$2$</td>
</tr>
<tr>
<td>$f$</td>
<td>$-$</td>
<td>$0$</td>
<td>$3$</td>
</tr>
</tbody>
</table>

C. Using your information in A & B, answer the following. Your answers must be consistent with A & B.

1. At what value(s) of $x$ does $f(x)$ have a local minimum? $x = 3$

2. On what interval(s) is $f(x)$ decreasing? $(-\infty, 3)$ or $(-\infty, 0) \cup (0, 3)$

3. At what value(s) of $x$ does $f(x)$ have a point of inflection? $x = 0, 2$

4. On what interval(s) is $f(x)$ concave down? $(0, 2)$