Show all work. Label all answers to word problems.

Q1: All questions refer to the complete graph of \( y = f(x) \) on \([-5, 5]\).

A. Find the following.
\[
\lim_{x \to 0^-} f(x) = -2 \quad \lim_{x \to 0^+} f(x) = 1 \quad f(0) = -2 \quad \lim_{x \to 2} f(x) = 1
\]

B. At what values of \( x \) does the limit of \( f(x) \) not exist? \( 0 \)

C. What is the slope of \( y = f(x) \) at \( x = -2 \)? \( -\frac{1}{3} \)

D. What is \( f'(1) \)? \( 0 \)

Q2: If \( f(x) \) is continuous at \( x = 3 \) and \( \lim_{x \to 3} f(x) = 17 \), for each statement determine if the statement may be true, must be true, or cannot be true. Circle your answer.

A. 3 is in the domain of \( f(x) \) may be true must be true cannot be true

B. \( \lim_{x \to 3} f(x) = 17 \) may be true must be true cannot be true

C. \( f(3) = 17 \) may be true must be true cannot be true

[Diagram]

\( f \) is continuous at \( x = 3 \) means

\[ f(3) \text{ exists} \]

\[ \lim_{x \to 3} f(x) \text{ exists} \]

\[ \lim_{x \to 3} f(x) = f(3) \]
Q3: Answer the questions about \( y = f(x) = \frac{2x^2 + 1}{x^2 - 4} \) and its graph, which is shown below. \( a \) and \( b \) are the locations of the vertical asymptotes.

\[
y-\text{intercept where } x = 0 : f(0) = \frac{0 + 1}{0 - 4} = -\frac{1}{4}
\]

A. What is the \( y - \) intercept? \(-\frac{1}{4}\)

\[
f(x) = \frac{2x^2 + 1}{x^2 - 4}
\]

B. What are the equations of the vertical asymptotes?

\[
\begin{align*}
  x^2 - 4 &= 0 \\
  (x-2)(x+2) &= 0
\end{align*}
\Rightarrow \quad \begin{align*}
  x - 2 &= 0 \quad \text{or} \quad x = 2 \\
  x + 2 &= 0 \quad \text{or} \quad x = -2
\end{align*}
\]

C. What is the equation of the horizontal asymptote?

\[
\lim_{x \to \infty} \frac{2x^2 + 1}{x^2 - 4} = \lim_{x \to \infty} \frac{2x^2}{x^2} = 2
\]

\[
y = 2
\]

D. \( \lim_{x \to b^+} f(x) = +\infty \)

E. \( \lim_{x \to b^-} f(x) = -\infty \)

Q4: Evaluate the limits.

A. \( \lim_{x \to -2} \frac{4x}{x^2 + x} = \frac{8}{6} = \frac{4}{3} \) \( \text{(2 pts)} \)

B. \( \lim_{x \to 0} \frac{4x}{x^2 + x} = \frac{0}{0} \text{ indeterminate} \)

\[
\lim_{x \to 0} \frac{4x}{x(x+1)} = \lim_{x \to 0} \frac{4}{x+1} = \frac{4}{1} = 4
\]

\( \text{(4 pts)} \)

C. \( \lim_{x \to +\infty} \frac{4x}{x^2 + x} = \lim_{x \to +\infty} \frac{4x}{x^2} = \lim_{x \to +\infty} \frac{4}{x} = 0 \)

D. \( \lim_{x \to -\infty} 4x^2 - 3x^5 + 16 \)

\[
\lim_{x \to -\infty} (-3x^5) = +\infty
\]

\( X \to -\infty \quad -(-)^5 \)
Q5: An object moves along a straight line so that its distance (in feet) from a fixed point at seconds after it begins to move is given by

\[ s(t) = t^2 + 2t + 1 \]

A. What is the object's average velocity between \( t = 1 \) and \( t = 5 \)?

\[
\frac{\Delta s}{\Delta t} = \frac{36 - 4}{5 - 1} = \frac{32}{4} = 8 \text{ ft/sec}
\]

B. What is the object's instantaneous velocity at \( t = 4 \) seconds?

Instantaneous velocity is the derivative (instantaneous rate of change).

\[ s'(t) = 2t + 2 \]

\[ s'(4) = 8 + 2 = 10 \text{ ft/sec} \]

Q6: Answer A & B about the function \( y = \frac{3}{x} \)

A. Find \( y' \)

\[ y = 3 \cdot x^{-1} \]

\[ y' = 3(-1) \cdot x^{-2} = -\frac{3}{x^2}, \text{ or } -3x^{-2} \]

B. Find the equation of the tangent line at \( x = 2 \) and sketch it on the graph.

\( \text{point} \; (2, f(2)) = (2, \frac{3}{2}) \)

\[ \text{slope} = f'(2) = -\frac{3}{4} \]

\[ \text{line} : \; y - y_0 = m(x-x_0) \]

Q7: A. Complete the illustration:
1. show the coordinates of the two points
2. draw the secant line between the two points

B. Complete the definition: The derivative of \( f \) at \( x \) is

\[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]

As \( h \to 0 \) slope of the secant line approaches the slope of the tangent line.
Q8: Suppose a person can memorize $y$ verses of a poem in $x$ hours as given by

$$y = f(x) = 5\sqrt{x} \quad 0 \leq x \leq 8$$

A. Find $f'(x)$

$$f(x) = 5x^{\frac{1}{2}}$$

$$f'(x) = \frac{5}{2} x^{-\frac{1}{2}}$$

B. $f(4) = 10$ and $f'(4) = 1.25$ Write a statement that interprets these values in terms of the word problem.

"After 4 hours, the person memorizes 10 verses of the poem and the number of verses he has memorized is increasing at a rate of 1.25 verses per hour."

C. Use differentials and the values given in part B to approximate the value of $f(4.5)$.

$$\Delta x = dx = 0.5$$

$$\Delta y = dy = f'(4) \cdot dx = 1.25 \cdot 0.5 = 1.25$$

$$f(4.5) \approx f(4) + dy = 10 + 1.25 = 11.25 \text{ verses}$$
Show all work. Label all answers to word problems.

Q1: All questions refer to the complete graph of $y = f(x)$ on $[-5, 5]$.

[Graph of a function with a slope of 2 at $x=2$]

A. Find the following.

$$\lim_{x \to 0^+} f(x) = \frac{1}{2}$$

$$\lim_{x \to 0^-} f(x) = -2$$

$$f(0) = -2$$

$$\lim_{x \to 2} f(x) = 1$$

B. At what values of $x$ does the limit of $f(x)$ not exist? $0$

C. What is the slope of $y = f(x)$ at $x = 3$? $2$

D. What is $f'(4)$? $0$

If $f(x)$ is continuous at $x = 3$, then

1. $f(3)$ exists (i.e., $3$ is in the domain of $f(x)$)
2. $\lim_{x \to 3} f(x)$ exists ($\because \lim_{x \to 3} f(x)$ must equal $\lim_{x \to 3} f(x) = 17$)
3. $\lim_{x \to 3} f(x) = f(3)$ (i.e., $f(3) = 17$)

Q2: If $f(x)$ is continuous at $x = 3$ and $\lim_{x \to 3} f(x) = 17$, for each statement determine if the statement must be true, may be true, or cannot be true. Circle your answer

A. $3$ is in the domain of $f(x)$ $\text{must be true}$ may be true cannot be true

B. $\lim_{x \to 3} f(x) = 17$ $\text{must be true}$ may be true cannot be true

C. $f(3) = 17$ $\text{must be true}$ may be true cannot be true
Q3: Answer the questions about \( y = f(x) = \frac{2x^2 + 1}{x^2 - 4} \) and its graph, which is shown below.

\( a \) and \( b \) are the locations of the vertical asymptotes.

\[ f(0) = \frac{0 + 1}{0 - 4} = -\frac{1}{4} \]

A. What is the \( y \)-intercept? \(-\frac{1}{4}\)

B. What are the equations of the vertical asymptotes?

\( x^2 - 4 = 0 \quad a \leftarrow x = 2, \quad x = -2 \)

\[ \boxed{X = 2 \quad X = -2} \]

C. What is the equation of the horizontal asymptote?

\[ \lim_{x \to \infty} \frac{2x^2 + 1}{x^2 - 4} = \lim_{x \to \infty} \frac{2x^2}{x^2} = 2 \]

\[ y = 2 \]

D. \( \lim_{x \to a^+} f(x) = -\infty \)

E. \( \lim_{x \to a^-} f(x) = +\infty \)

Q4: Evaluate the limits.

A. \( \lim_{x \to 2} \frac{x^2 + x}{4x} = \frac{4 + 2}{8} = \frac{3}{4} \)

B. \( \lim_{x \to 0} \frac{x^2 + x}{4x} = \frac{0}{0} \text{ in indeterminate} \Rightarrow \text{factor} \)

\[ \lim_{x \to 0} \frac{x(x+1)}{4x} = \lim_{x \to 0} \frac{1}{4} = \frac{1}{4} \]

C. \( \lim_{x \to \infty} \frac{x^2 + x}{4x} = \lim_{x \to \infty} \frac{x^2}{4x} = \lim_{x \to \infty} \frac{x}{4} = +\infty \]

D. \( \lim_{x \to \infty} 4x^2 - 3x^5 + 16 \)

\( \lim_{x \to \infty} -3x^5 = -\infty \quad \lim_{x \to \infty} -3(\infty) = \text{negative} \)
Q5: An object moves along a straight line so that its distance (in feet) from a fixed point \( t \) seconds after it begins to move is given by

\[ s(t) = t^2 + 4t + 10 \]

A. What is the object's average velocity between \( t = 1 \) and \( t = 5 \)?

\[ S(1) = 1 + 4 + 10 = 15 \]
\[ S(5) = 25 + 26 + 10 = 55 \]
\[ \Delta s = \frac{55 - 15}{5 - 1} = \frac{40}{4} = 10 \text{ ft/sec} \]

B. What is the object's instantaneous velocity at \( t = 4 \) seconds?

\[ S'(t) \text{ is instantaneous rate of change or instantaneous velocity} \]
\[ S'(t) = 2t + 4 \]
\[ S'(4) = 8 + 4 = 12 \text{ ft/sec} \]

Q6: Answer A & B about the function \( y = \frac{3}{x} \)

A. Find \( y' \)

\[ y = 3x^{-1} \]
\[ y' = -3x^{-2} \]
\[ y' = \frac{-3}{x^2} = \frac{-3}{x^2} \]

B. Find the equation of the tangent line at \( x = 3 \) and sketch it on the graph.

\[ m = \text{point } (3, f(3)) = (3, 1) \]
\[ \text{Slope } S'(3) = \frac{-3}{3^2} = \frac{-1}{3} \]
\[ \text{Line: } y - y_0 = m(x - x_0) \]

Q7: A. Complete the illustration:

1. show the coordinates of the two points
2. draw the secant line between the two points

B. Complete the definition: The derivative of \( f \) at \( x \) is

\[ f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \]
\[ \text{slope of secant line as } h \to 0 \leftarrow \text{slope of tangent} \]
Q8: Suppose a person can memorize $y$ verses of a poem in $x$ hours as given by

$$y = f(x) = 5\sqrt{x} \quad 0 \leq x \leq 8$$

A. Find $f'(x)$

$$y' = f'(x) = \frac{5x}{2}$$

$$f'(x) = 5\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$$

$$= \frac{5}{2}x^{-\frac{1}{2}}$$

B. $f(4) = 10$ and $f'(4) = 1.25$ Write a statement that interprets these values in terms of the word problem.

*After 4 hours, 10 verses have been memorized and the number of memorized verses is increasing at a rate of 1.25 verses per hour.*

C. Use differentials and the values given in part B to approximate the value of $f(4.6)$.

$$\Delta x = dx = .6$$

$$\Delta y \approx dy = f'(4) \cdot dx = 1.25 \cdot (.6) = .750$$

$$f(4.6) \approx f(4) + dy = 10 + .750 = 10.75$$
Show all work. Label all answers to word problems.

**Q1:** All questions refer to the complete graph of \( y = f(x) \) on \([-5, 5]\).

A. Find the following.

\[
\lim_{x \to 0^{-}} f(x) = \quad \lim_{x \to 0^{+}} f(x) = \quad f(0) = \quad \lim_{x \to 2} f(x) =
\]

B. At what values of \( x \) does the limit of \( f(x) \) not exist? ________________

C. What is the slope of \( y = f(x) \) at \( x = -2 \)? ________________

D. What is \( f'(1) \)? ________________

**Q2:** If \( f(x) \) is continuous at \( x = 3 \) and \( \lim_{x \to 3} f(x) = 17 \), for each statement determine if the statement may be true, must be true, or cannot be true. Circle your answer

A. 3 is in the domain of \( f(x) \) may be true must be true cannot be true

B. \( \lim_{x \to 3} f(x) = 17 \) may be true must be true cannot be true

C. \( f(3) = 17 \) may be true must be true cannot be true
Q3: Answer the questions about \( y = f(x) = \frac{2x^2 + 1}{x^2 - 4} \) and its graph, which is shown below. \( a \) and \( b \) are the locations of the vertical asymptotes.

A. What is the \( y \) - intercept? ________

B. What are the equations of the vertical asymptotes?

C. What is the equation of the horizontal asymptote?

\[
f(x) = \frac{2x^2 + 1}{x^2 - 4}
\]

D. \( \lim_{x \to b^+} f(x) = \) ________

E. \( \lim_{x \to b^-} f(x) = \) ________

Q4: Evaluate the limits.

A. \( \lim_{x \to 2} \frac{4x}{x^2 + x} \)

B. \( \lim_{x \to 0} \frac{4x}{x^2 + x} \)

C. \( \lim_{x \to +\infty} \frac{4x}{x^2 + x} \)

D. \( \lim_{x \to -\infty} 4x^2 - 3x^5 + 16 \)
Q5: An object moves along a straight line so that its distance (in feet) from a fixed point $t$ seconds after it begins to move is given by

$$s(t) = t^2 + 2t + 1$$

A. What is the object's average velocity between $t = 1$ and $t = 5$?

B. What is the object's instantaneous velocity at $t = 4$ seconds?

Q6: Answer A & B about the function $y = \frac{3}{x}$

A. Find $y'$

B. Find the equation of the tangent line at $x = 2$ and sketch it on the graph.

Q7: A. Complete the illustration:
   1. show the coordinates of the two points
   2. draw the secant line between the two points

B. Complete the definition: The derivative of $f$ at $x$ is

$$f'(x) =$$
Q8: Suppose a person can memorize $y$ verses of a poem in $x$ hours as given by

$$ y = f(x) = 5\sqrt{x} \quad 0 \leq x \leq 8 $$

A. Find $f'(x)$

B. $f(4) = 10$ and $f'(4) = 1.25$ Write a statement that interprets these values in terms of the word problem.

C. Use differentials and the values given in part B to approximate the value of $f(4.5)$.

$$ \Delta x = dx = \underline{\hspace{2cm}} $$

$$ \Delta y \approx dy = \underline{\hspace{2cm}} $$

$$ f(4.5) \approx \underline{\hspace{2cm}} $$
Show all work. Label all answers to word problems.

Q1: All questions refer to the complete graph of $y = f(x)$ on $[-5, 5]$.

A. Find the following.
\[ \lim_{x \to 0^+} f(x) = \quad \lim_{x \to 0^-} f(x) = \quad f(0) = \quad \lim_{x \to 2} f(x) = \]

B. At what values of $x$ does the limit of $f(x)$ not exist?

C. What is the slope of $y = f(x)$ at $x = 3$?

D. What is $f'(-4)$?

Q2: If $f(x)$ is continuous at $x = 3$ and $\lim_{x \to 3^-} f(x) = 17$, for each statement determine if the statement must be true, may be true, or cannot be true. Circle your answer

A. $3$ is in the domain of $f(x)$  
   must be true  
   may be true  
   cannot be true

B. $\lim_{x \to 3} f(x) = 17$  
   must be true  
   may be true  
   cannot be true

C. $f(3) = 17$  
   must be true  
   may be true  
   cannot be true
Q3: Answer the questions about \( y = f(x) = \frac{2x^2 + 1}{x^2 - 4} \) and its graph, which is shown below. \( a \) and \( b \) are the locations of the vertical asymptotes.

A. What is the \( y \)-intercept? 

\[ f(x) = \frac{2x^2 + 1}{x^2 - 4} \]

B. What are the equations of the vertical asymptotes?

C. What is the equation of the horizontal asymptote?

D. \( \lim_{x \to a^+} f(x) = \) 

E. \( \lim_{x \to a^-} f(x) = \)

Q4: Evaluate the limits.

A. \( \lim_{x \to 2} \frac{x^2 + x}{4x} \)

B. \( \lim_{x \to 0} \frac{x^2 + x}{4x} \)

C. \( \lim_{x \to +\infty} \frac{x^2 + x}{4x} \)

D. \( \lim_{x \to +\infty} 4x^2 - 3x^5 + 16 \)
Q5: An object moves along a straight line so that its distance (in feet) from a fixed point $t$ seconds after it begins to move is given by 

$$s(t) = t^2 + 4t + 10$$

A. What is the object's average velocity between $t = 1$ and $t = 5$?

B. What is the object's instantaneous velocity at $t = 4$ seconds?

Q6: Answer A & B about the function $y = \frac{3}{x}$

A. Find $y'$

B. Find the equation of the tangent line at $x = 3$ and sketch it on the graph.

Q7: A. Complete the illustration:
   1. show the coordinates of the two points
   2. draw the secant line between the two points

B. Complete the definition: The derivative of $f$ at $x$ is 

$$f'(x) =$$
Q8: Suppose a person can memorize $y$ verses of a poem in $x$ hours as given by

$$ y = f(x) = 5\sqrt{x} \quad 0 \leq x \leq 8 $$

A. Find $f'(x)$

B. $f(4) = 10$ and $f'(4) = 1.25$ Write a statement that interprets these values in terms of the word problem.

C. Use differentials and the values given in part B to approximate the value of $f(4.6)$.

$\Delta x = dx = \underline{\hspace{2cm}}$

$\Delta y \approx dy = \underline{\hspace{2cm}}$

$f(4.6) \approx \underline{\hspace{2cm}}$