Q1: Each augmented matrix has been row reduced. Either
- find the unique solution
- find that there is no solution
- find the infinite solutions using a parameter \( t \) and list the possible values of \( t \) that guarantee that the \( x \) values are non-negative, whole numbers.

A. \[
\begin{bmatrix}
1 & 0 & -1 & | -4 \\
0 & 1 & 2 & | 15 \\
0 & 0 & 0 & | 0
\end{bmatrix}
\]

\[
x_1 = \frac{t - 4}{-2} \\
x_2 = \frac{-2t + 15}{3} \\
x_3 = \frac{t}{1}
\]

\[
\begin{align*}
t - 4 & \geq 0 \\
-2t + 15 & \geq 0 \\
t & \leq 7.5
\end{align*}
\]

\[
\Rightarrow t = 4, 5, 6, 7
\]

B. \[
\begin{bmatrix}
1 & 0 & 1 & | 3 \\
0 & 1 & 3 & | 2 \\
0 & 0 & 0 & | 1
\end{bmatrix}
\]

\[
x_1 = \\
x_2 = \\
x_3 = \text{no solution}
\]

Q2: A. Complete the multiplication
\[
\begin{bmatrix}
a & 1 \\
2 & 3
\end{bmatrix}
\begin{bmatrix}
4 & -2 \\
1 & b
\end{bmatrix} = 
\begin{bmatrix}
4a + 1 & -2a + b \\
2 & 4 + 3b
\end{bmatrix}
\]

B. \[
M = \begin{bmatrix}
23 & 14 & 77 & 12 \\
33 & 45 & 65 & 42 \\
56 & 19 & 51 & 38
\end{bmatrix}
\]

\[
A = \begin{bmatrix}
1 \\
1 \\
1
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
1 \\
1 \\
1
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
1 & 1 & 1
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
1 & 1 & 1 & 1
\end{bmatrix}
\]

a. Which matrix multiplication will result in a matrix that stores the sums of the numbers in each of the columns of \( M \)? Circle one: \( \text{AM, BM, CM, DM, MA, MB, MC, MD} \)

\(\text{CM, only MB and CM are legal}\)

b. What are the dimensions of the product matrix that you circled above? \( 1 \times 4 \) row matrix

c. What matrix results from the multiplication \( CA \)?
\[
\begin{bmatrix}
1 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
1 \\
1 \\
1
\end{bmatrix} = \begin{bmatrix}
3
\end{bmatrix}
\]
Q3: Solve these matrix equations for $X$

A. $AX - B = CX$
\[
AX + CX = B
\]
\[
(A-C)X = B
\]
\[
X = (A-C)^{-1}B
\]

B. For partial credit, show intermediate steps. Calculations may be completed with your calculator.
\[
\begin{bmatrix}
3 & 1 & x_1 \\
2 & 1 & x_2 \\
\end{bmatrix} + \begin{bmatrix}
-4 \\
1 \\
\end{bmatrix} = \begin{bmatrix}
7 \\
3 \\
\end{bmatrix}
\]
\[
\begin{bmatrix}
3 & 1 \\
2 & 1 \\
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2 \\
\end{bmatrix} = \begin{bmatrix}
7 \\
3 \\
\end{bmatrix}
\]
\[
\begin{bmatrix}
3 & 1 \\
2 & 1 \\
\end{bmatrix}^{-1} \begin{bmatrix}
x_1 \\
x_2 \\
\end{bmatrix} = \begin{bmatrix}
7 \\
3 \\
\end{bmatrix}
\]
\[
\begin{bmatrix}
1 & -1 & x_1 \\
-2 & 3 & x_2 \\
\end{bmatrix} \begin{bmatrix}
1 \\
1 \\
\end{bmatrix} = \begin{bmatrix}
9 \\
-16 \\
\end{bmatrix}
\]

Q4: For an upcoming concert an outdoor amphitheater is selling tickets at prices of $8, $12 and $20. We know that the total number of tickets is 25,000, that the number of $8 tickets is the same as the number of $20 tickets and that the required revenue must be $320,000. How many of each type of ticket should be sold?

A. Define the variables and set up the system of equations.
\[
\begin{align*}
\chi_1 &= \text{ # of}$8 tickets \\
\chi_2 &= \text{ # of}$12 tickets \\
\chi_3 &= \text{ # of}$20 tickets \\
\chi_1 + \chi_2 + \chi_3 &= 25,000 \\
\chi_1 - \chi_3 &= 0 \quad (\chi_1 = \chi_3) \\
8\chi_1 + 12\chi_2 + 20\chi_3 &= 320,000
\end{align*}
\]

B. Set up the augmented matrix and solve using your calculator.
\[
\begin{bmatrix}
1 & 1 & 1 & | & 25,000 \\
1 & 0 & -1 & | & 0 \\
8 & 12 & 20 & | & 320,000
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 0 & | & 5,000 \\
0 & 1 & 0 & | & 15,000 \\
0 & 0 & 1 & | & 5,000
\end{bmatrix}
\]

$5,000$ - $8$ and $20$ tickets

$15,000$ - $12$ tickets
Q5: Leontief Input-Output Analysis: The technology matrix for an economy based on two sectors, transportation (T) and electricity (E) is

\[
M = \begin{bmatrix}
T & E \\
0.4 & 0.1 \\
0.2 & 0.3 \\
\end{bmatrix}
\]

The final demand of the economy is $15 billion for transportation and $20 billion for electricity.

A. The production of a dollar's worth of energy requires \(0.2\) of input from the transportation sector, and \(0.3\) of input from the energy sector.

B. What is the matrix equation of the system of equations that you want to solve?

\[
X = M X + D
\]

C. The solution of the matrix equation is \(X = (I - M)^{-1}D\),

Find the final output matrix \(X\), showing the intermediate matrices as you use them. Calculations may be completed with a calculator.

\[
I - M = \begin{bmatrix}
1 & 0 \\
0 & 1 \\
\end{bmatrix} - \begin{bmatrix}
0.4 & 0.1 \\
0.2 & 0.3 \\
\end{bmatrix} = \begin{bmatrix}
0.6 & -0.1 \\
-0.2 & 0.7 \\
\end{bmatrix}
\]

\[
(I - M)^{-1} = \begin{bmatrix}
1.75 & -0.25 \\
-0.5 & 1.5 \\
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
15 \\
20 \\
\end{bmatrix}
\]

\[
X = \begin{bmatrix}
31.25 \\
37.5 \\
\end{bmatrix}
\]

The final output is $31.25 billion of transportation and $37.5 billion of energy.

D. Write a statement that interprets the answer you found above.
Q6: Find the maximum and minimum values of the function

\[ P = 20x_1 + 15x_2 \text{ subject to the constraints } \begin{cases} 2x_1 + 3x_2 \leq 30 \quad \text{A} \\ 2x_1 + x_2 \leq 26 \quad \text{B} \\ x_1, x_2 \geq 0 \end{cases} \]

A. Shade in the feasible region on the graph below and show your work in determining the maximum value of \( P \).

\[ \begin{array}{c|c|c}
\text{Point} & \text{Calculation} & \text{Value} \\
\hline
(0,0) & 0 + 0 = 0 & 0 \\
(0,10) & 0 + 150 = 150 & 150 \\
(13,0) & 260 + 0 = 260 & 260 \\
(12,2) & 240 + 30 = 270 & 270 \\
\end{array} \]

The maximum value of \( P \) is \( \underline{270} \) at \((x,y) = (12,2)\).

The minimum value of \( P \) is \( \underline{0} \) at \((x,y) = (0,0)\).

B. Construct the initial simplex tableau for solving this standard maximization problem. Label the columns.

\[ \begin{align*}
2x_1 + 3x_2 &\leq 30 \\
2x_1 + x_2 &\leq 26 \\
-20x_1 - 15x_2 + P &= 0
\end{align*} \]

\[ \begin{bmatrix}
x_1 & x_2 & s_1 & s_2 & P \\ 2 & 3 & 1 & 0 & 0 & 30 \\
2 & 1 & 0 & 1 & 0 & 26 \\
-20 & -15 & 0 & 0 & 1 & 0
\end{bmatrix} \]

C. Circle the first pivot element in the simplex tableau above.
Q7: A. The first pivot operations were performed on the correct simplex tableau from Q6. What is the current basic solution?

\[
\begin{bmatrix}
 x_1 & x_2 & s_1 & s_2 & P \\
 \hline
 s_1 & 0 & 2 & 1 & -1 & 0 & 4 \\
 s_2 & 1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 13 \\
 \rho & 0 & -5 & 0 & 10 & 1 & 260
\end{bmatrix}
\]

Basic Solution:

\[
\begin{align*}
x_1 &= \frac{1}{3} \\
x_2 &= 0 \\
s_1 &= 4 \\
s_2 &= 0 \\
P &= 260 \tag{13, 0}
\end{align*}
\]

B. Using the simplex tableau and the circled pivot element, complete the pivot operations. List the row operations that you use.

\[
\begin{bmatrix}
 x_1 & x_2 & s_1 & s_2 & P \\
 \hline
 0 & 2 & 1 & -1 & 0 & 4 \\
 1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 13 \\
 0 & -5 & 0 & 10 & 1 & 260
\end{bmatrix}
\]

\[
\begin{align*}
\frac{1}{2}R_1 \rightarrow R_1: & \quad \begin{bmatrix} 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 & 2 \\ \frac{1}{2} & 0 & 1 & 0 & 13 \\ 0 & -5 & 0 & 10 & 1 & 260 \end{bmatrix} \\
\end{align*}
\]

\[
\begin{align*}
-\frac{1}{2}R_1 + R_2 \rightarrow R_2: & \quad \begin{bmatrix} 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 & 2 \\ 1 & 0 & -\frac{1}{2} & \frac{3}{4} & 0 & 12 \\ 0 & -5 & 0 & 10 & 1 & 260 \end{bmatrix} \\
5R_1 + R_3 \rightarrow R_3: & \quad \begin{bmatrix} 0 & 0 & \frac{5}{2} & \frac{15}{2} & 1 & 270 \\ \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\chi_1 &= 12 \\
x_2 &= 2 \\
s_1 &= 0 \\
s_2 &= 0 \\
P &= 270 \tag{12, 2}
\end{align*}
\]
Q1: Each augmented matrix has been row reduced. Either
- find the unique solution
- find that there is no solution
- find the infinite solutions using a parameter \( t \) and list the possible values of \( t \) that guarantee that the \( x \) values are non-negative, whole numbers.

A. \[
\begin{bmatrix}
1 & 0 & -1 & -3 \\
0 & 1 & 2 & 13 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]
\[
x_1 = t - 3 \\
x_2 = \frac{-2t + 13}{13} \\
x_3 = t
\]
\[
t \geq 3 \\
t \geq -2 \\
t \leq \frac{13}{3} = 4.33
\]
\[
\therefore t = 3, 4, 5, 6
\]

B. \[
\begin{bmatrix}
1 & 0 & 1 & 3 \\
0 & 1 & 3 & 2 \\
0 & 0 & 0 & 5
\end{bmatrix}
\]
\[
x_1 = -
\]
x_2 = -
\]
x_3 = no solution.

Q2: A. Complete the multiplication
\[
\begin{bmatrix}
2 & 1 \\
3 & a
\end{bmatrix}
\begin{bmatrix}
b & -2
\end{bmatrix}
= \begin{bmatrix}
2b + 1 & -1 \\
3b + a & -6 + 3a
\end{bmatrix}
\]
\[
3 \times 4 \quad 3 \times 1 \quad 4 \times 1
\]

B. \[
M = \begin{bmatrix}
23 & 14 & 77 & 12 \\
33 & 45 & 65 & 42 \\
56 & 19 & 51 & 38
\end{bmatrix}
\]
\[
A = \begin{bmatrix}
1 \\
1 \\
1
\end{bmatrix}
\]
\[
B = \begin{bmatrix}
1 \\
1 \\
1
\end{bmatrix}
\]
\[
C = \begin{bmatrix}
1 & 1 & 1
\end{bmatrix}
\]
\[
D = \begin{bmatrix}
1 & 1 & 1 & 1
\end{bmatrix}
\]

a. Which matrix multiplication will result in a matrix that stores the sums of the numbers in each of the rows of \( M \)? Circle one:

AM  BM  CM  DM  MA  MB  MC  MD

only MB & CM are legal

b. What are the dimensions of the product matrix that you circled above? 3 \times 1 column matrix

c. What matrix results from the multiplication \( DB \)?
\[
\begin{bmatrix}
1 & 1 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
1 \\
1 \\
1 \\
1
\end{bmatrix}
= \begin{bmatrix}
4
\end{bmatrix}
\]
Q3: Solve these matrix equations for $X$

A. $CX - A = BX$

$$\begin{align*}
C \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= B \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + A \\
C \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= A \\
(C - B) \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= A \\
\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= (C - B)^{-1} A
\end{align*}$$

B. For partial credit, show intermediate steps. Calculations may be completed with your calculator.

$$\begin{align*}
\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= \begin{bmatrix} 7 \\ 3 \end{bmatrix} + \begin{bmatrix} -4 \\ 1 \end{bmatrix} \\
\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= \begin{bmatrix} 7 \\ 3 \end{bmatrix} \\
\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 7 \\ 3 \end{bmatrix} \\
&= \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \end{bmatrix} \\
&= \begin{bmatrix} 10 \\ 6 \end{bmatrix}
\end{align*}$$

Q4: For an upcoming concert an outdoor amphitheater is selling tickets at prices of $8, $12 and $20. We know that the total number of tickets is 25,000, that the number of $8 tickets is the same as the number of $20 tickets and that the required revenue must be $340,000. How many of each type of ticket should be sold?

A. Define the variables and set up the system of equations.

$$\begin{align*}
X_1 &= \# \text{ of } \$8 \text{ tickets} \\
X_2 &= \# \text{ of } \$12 \text{ tickets} \\
X_3 &= \# \text{ of } \$20 \text{ tickets}
\end{align*}$$

$$X_1 + X_2 + X_3 = 25,000$$

$$X_1 - X_3 = 0 \quad (X_1 = X_3)$$

$$8X_1 + 12X_2 + 20X_3 = 340,000$$

B. Set up the augmented matrix and solve using your calculator.

$$\begin{bmatrix}
1 & 1 & 1 & 25,000 \\
1 & 0 & -1 & 0 \\
8 & 12 & 20 & 340,000
\end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix}
1 & 0 & 0 & 10,000 \\
0 & 1 & 0 & 5,000 \\
0 & 0 & 1 & 10,000
\end{bmatrix}$$

10,000 - $8 and $20 tickets

5,000 - $12 tickets
Q5: Leontief Input-Output Analysis: The technology matrix for an economy based on two sectors, transportation (T) and electricity (E) is

\[
M = \begin{bmatrix}
T & E \\
0.4 & 0.1 \\
0.2 & 0.3
\end{bmatrix}
\]

The final demand of the economy is $15 billion for transportation and $20 billion for electricity.

A. The production of a dollar's worth of transportation requires \( \frac{3}{4} \) of input from the transportation sector, and \( \frac{1}{2} \) of input from the energy sector.

B. What is the matrix equation of the system of equations that you want to solve?

\[
X = MX + D
\]

C. The solution of the matrix equation is \( X = (I - M)^{-1}D \),

Find the final output matrix \( X \), showing the intermediate matrices as you use them. Calculations may be completed with a calculator.

\[
I - M = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} - \begin{bmatrix}
0.4 & 0.1 \\
0.2 & 0.3
\end{bmatrix} = \begin{bmatrix}
0.6 & -0.1 \\
-0.2 & 0.7
\end{bmatrix}
\]

\[
(I - M)^{-1}D = \begin{bmatrix}
1.75 & -25 \\
0.5 & 15
\end{bmatrix} \begin{bmatrix}
15 \\
20
\end{bmatrix}
\]

\[
X = \begin{bmatrix}
31.25 \\
37.5
\end{bmatrix}
\]

D. Write a statement that interprets the answer you found above.

The final output of transportation is \$31.25 billion

And of energy is \$37.5 billion
Q6: Find the maximum and minimum values of the function

\[ P = 20x_1 + 15x_2 \text{ subject to the constraints } \begin{align*}
2x_1 + 3x_2 & \leq 30 \quad A \\
2x_1 + x_2 & \leq 26 \quad B \\
x_1, x_2 & \geq 0
\end{align*} \]

A. Shade in the feasible region on the graph below and show your work in determining the maximum value of \( P \).

\[ \begin{array}{|c|}
\hline
(x, y) & P = 20x_1 + 15x_2 \\
\hline
(0, 0) & 0 + 0 = 0 \\
(0, 10) & 0 + 150 = 150 \\
(13, 0) & 260 + 0 = 260 \\
(12, 2) & 240 + 30 = 270 \\
\hline
\end{array} \]

The maximum value of \( P \) is \( 270 \) at \((x, y) = (12, 2)\).

The minimum value of \( P \) is \( 0 \) at \((x, y) = (0, 0)\).

B. Construct the initial simplex tableau for solving this standard maximization problem. Label the columns.

\[
\begin{align*}
2x_1 + 3x_2 & \leq 30 \\
2x_1 + x_2 & \leq 26 \\
-20x_1 - 15x_2 + P &= 0
\end{align*}
\]

\[
\begin{bmatrix}
2 & 3 & 1 & 0 & 0 & \frac{30}{2} = 15 \\
2 & 1 & 0 & 1 & 0 & \frac{26}{2} = 13 \\
-20 & -15 & 0 & 0 & 1
\end{bmatrix}
\]

C. Circle the first pivot element in the simplex tableau above.
Q7: A. The first pivot operations were performed on the correct simplex tableau from Q6. What is the current basic solution?

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>0</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>-5</td>
<td>0</td>
<td>10</td>
<td>1</td>
</tr>
</tbody>
</table>

Basic Solution:

$x_1 = \frac{13}{13}$  \quad $x_2 = 0$

$s_1 = \frac{4}{13}$  \quad $s_2 = 0$

$P = 260 \quad @ \quad (13, 0)$

B. Using the simplex tableau and the circled pivot element, complete the pivot operations. List the row operations that you use.

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>0</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>-5</td>
<td>0</td>
<td>10</td>
<td>1</td>
</tr>
</tbody>
</table>

$\frac{1}{2}R_1 \rightarrow R_1$

\[
\begin{bmatrix}
0 & 1 & 0.5 & -0.5 & 0 & \frac{2}{0} \\
0 & 0 & 0 & 0 & 1 & 13 \\
0 & -5 & 0 & 10 & 1 & 260
\end{bmatrix}
\]

$\frac{1}{2}R_1 + R_2 \rightarrow R_2$

$5R_1 + R_3 \rightarrow R_3$

$\chi_1 = 12$  \quad $x_2 = 2$

$s_1 = 0$  \quad $s_2 = 0$

$P = 270 \quad @ \quad (12, 2)$
Q1: Each augmented matrix has been row reduced. Either
- find the unique solution
- find that there is no solution
- find the infinite solutions using a parameter $t$ and list the possible values of $t$ that guarantee that the $x$ values are non-negative, whole numbers.

A. $\begin{bmatrix} 1 & 0 & -1 & | & -4 \\ 0 & 1 & 2 & | & 15 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$

$x_1 =$

$x_2 =$

$x_3 =$

B. $\begin{bmatrix} 1 & 0 & 1 & | & 3 \\ 0 & 1 & 3 & | & 2 \\ 0 & 0 & 0 & | & 1 \end{bmatrix}$

$x_1 =$

$x_2 =$

$x_3 =$

Q2: A. Complete the multiplication

$\begin{bmatrix} a & 1 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & -2 \\ 1 & b \end{bmatrix} =$

B. $M = \begin{bmatrix} 23 & 14 & 77 & 12 \\ 33 & 45 & 65 & 42 \\ 56 & 19 & 51 & 38 \end{bmatrix}$

$A = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$C = [1 \ 1 \ 1]$  

$D = [1 \ 1 \ 1 \ 1]$

a. Which matrix multiplication will result in a matrix that stores the sums of the numbers in each of the columns of $M$? Circle one:

AM  BM  CM  DM  MA  MB  MC  MD

b. What are the dimensions of the product matrix that you circled above? ________

c. What matrix results from the multiplication $CA$? 
Q3: Solve these matrix equations for $X$

A. $AX - B = CX$

B. For partial credit, show intermediate steps. Calculations may be completed with your calculator.

$$\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -4 \\ -1 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

Q4: For an upcoming concert an outdoor amphitheater is selling tickets at prices of $8, $12 and $20. We know that the total number of tickets is 25,000, that the number of $8 tickets is the same as the number of $20 tickets and that the required revenue must be $320,000. How many of each type of ticket should be sold?

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\[
M = \begin{bmatrix}
T & E \\
0.4 & 0.1 \\
0.2 & 0.3
\end{bmatrix}
\]

The final demand of the economy is $15 billion for transportation and $20 billion for electricity.

A. The production of a dollar's worth of energy requires _____ of input from the transportation sector, and _____ of input from the energy sector.

B. What is the matrix equation of the system of equations that you want to solve?

C. The solution of the matrix equation is \( X = (I - M)^{-1} D \),

Find the final output matrix \( X \), showing the intermediate matrices as you use them. Calculations may be completed with a calculator.

D. Write a statement that interprets the answer you found above.
Q6: Find the maximum and minimum values of the function

\[ P = 20x_1 + 15x_2 \]

subject to the constraints

\[
\begin{align*}
2x_1 + 3x_2 &\leq 30 \\
2x_1 + x_2 &\leq 26 \\
x_1, x_2 &\geq 0
\end{align*}
\]

A. Shade in the feasible region on the graph below and show your work in determining the maximum value of \( P \). 

The maximum value of \( P \) is \( \text{___________} \) at \( (x_1, x_2) = \text{___________} \)

The minimum value of \( P \) is \( \text{___________} \) at \( (x_1, x_2) = \text{___________} \)

B. Construct the initial simplex tableau for solving this standard maximization problem. Label the columns.

C. Circle the first pivot element in the simplex tableau above.
Q7:  A. The first pivot operations were performed on the correct simplex tableau from Q6. What is the current basic solution?

\[
\begin{array}{ccccc}
  x_1 & x_2 & s_1 & s_2 & P \\
  \hline
  0 & 2 & 1 & -1 & 0 & 4 \\
  1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 13 \\
  0 & -5 & 0 & 10 & 1 & 260 \\
\end{array}
\]

Basic Solution:

\[
\begin{align*}
  x_1 &= \_\_\_\_\_ \quad x_2 &= \_\_\_\_\_ \\
  s_1 &= \_\_\_\_\_ \quad s_2 &= \_\_\_\_\_ \\
  P &= \_\_\_\_\_ @ \_\_\_\_\_ \\
\end{align*}
\]

B. Using the simplex tableau and the circled pivot element, complete the pivot operations. List the row operations that you use.

\[
\begin{array}{ccccc}
  x_1 & x_2 & s_1 & s_2 & P \\
  \hline
  0 & \_\_\_\_\_ & 1 & -1 & 0 & 4 \\
  1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 13 \\
  0 & -5 & 0 & 10 & 1 & 260 \\
\end{array}
\]