Math 1390 – Test #3A   Score: ____

Spring 2010   Name Solution A   80

Q1: A trucking firm wants to purchase 10 trucks that will provide exactly 28 tons of additional shipping capacity. A model A truck holds 2 tons, a model B truck holds 3 tons, and a model C truck holds 5 tons. How many trucks of each model should the company purchase to provide the additional shipping capacity?

A. If x_1, x_2 and x_3 represent the number of model A, B and C trucks, respectively, set up the system of linear equations and the augmented matrix that are used to solve this problem.

$$x_1 + x_2 + x_3 = 10$$
$$2x_1 + 3x_2 + 5x_3 = 28$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 10 \\ 2 & 3 & 5 & 28 \end{array}\right]$$

B. When the correct matrix for the problem is row-reduced we get the matrix below. Introduce a parameter t to write the possible solutions of the problem. Recognizing that we are talking about the number of trucks, list the possible values of t.

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 2 \\ 0 & 1 & 3 & 8 \end{array}\right]$$

$x_1 =$ $2t + 2$ $\Rightarrow 2t + 2 \geq 0 \Rightarrow 2t \geq -2 \Rightarrow \boxed{t \geq -1}$

$x_2 =$ $-3t + 8$ $\Rightarrow -3t + 8 \geq 0 \Rightarrow -3t \geq -8 \Rightarrow \boxed{t \leq \frac{8}{3}}$ Note $\frac{8}{3} = 2\frac{2}{3}$

$x_3 =$ t $\Rightarrow \boxed{t \geq 0}$ where t can equal $t = 0, 1, 2$

$0 \leq t \leq \frac{8}{3}$, t is a whole # $\therefore$

C. If the purchase prices (in thousands of \$) of the truck models A, B and C are 20, 40, and 50, respectively, what is the total cost of the trucks when $t = 1$?

$t = 1 \Rightarrow \quad x_1 = 4, \quad x_2 = 5, \quad x_3 = 1$

$\therefore$ total cost $C = 20x_1 + 40x_2 + 50x_3$

$C = 20(4) + 40(5) + 50(1)$

$C = 80 + 200 + 50$

$C = 330$ or $\boxed{\$330{,}000}$

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Q2: A company makes three chocolate candies: cherry, almond, and raisin. Matrix A gives the number of units of each ingredient, sugar, chocolate and milk, in a batch of each type of candy. Matrix B gives the cost of one unit of each ingredient, in dollars, from suppliers X and Y.

$$A = \begin{array}{r} \\ cherry \\ almond \\ raisin \end{array} \begin{array}{c} \begin{array}{ccc} sugar & choc & milk \end{array} \\ \begin{bmatrix} 4 & 6 & 1 \\ 5 & 3 & 1 \\ 3 & 3 & 1 \end{bmatrix} \end{array} \qquad B = \begin{array}{r} \\ sugar \\ choc \\ milk \end{array} \begin{array}{c} \begin{array}{cc} X & Y \end{array} \\ \begin{bmatrix} 3 & 2 \\ 3 & 4 \\ 2 & 2 \end{bmatrix} \end{array}$$

A. The product of A and B is the matrix $C = \begin{bmatrix} 32 & 34 \\ 26 & 24 \\ 20 & 20 \end{bmatrix}$

(Circle one answer) What does the 34 in this matrix represent?

Using supplier Y:

1. the total cost of sugar in making one batch of each candy
2. the total cost of chocolate in making one batch of each candy
3. the total cost of milk in making one batch of each candy
4. the total cost of all ingredients for making one batch of cherry candy
5. the total cost of all ingredients for making one batch of almond candy
6. the total cost of all ingredients for making one batch of raisin candy

Use the matrices C and $D - G$ to answer the following questions.

3X2 1X2 1×3 2×1 3×1

$$C = \begin{bmatrix} 32 & 34 \\ 26 & 24 \\ 20 & 20 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 1 \end{bmatrix} \qquad E = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \qquad F = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad G = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

B. Which two of the following multiplications of the matrix C are legal? Circle your answers.

DC $\quad EC$ $\quad FC$ $\quad GC$ $\quad CD$ $\quad CE$ $\quad CF$ $\quad CG$

C. If we want to use a matrix multiplication to find the total sum of the elements of each column in matrix C, which matrix multiplication that you circled above should be used? answer: EC

10

Q3: A. Circle: Solve for the matrix X in the matrix equation $RX = SX + T$. $X =$

1. $T(R-S)^{-1}$ 2. $\dfrac{T}{R-S}$ 3. $(R-S)^{-1}T$ 4. $\dfrac{T}{I-RS}$ 5. $T(I-RS)^{-1}$ 6. $(I-RS)^{-1}T$

$RX - SX = T \Rightarrow (R-S)X = T \Rightarrow (R-S)^{-1}(R-S)X = (R-S)^{-1}T \Rightarrow \boxed{X = (R-S)^{-1}T}$

B. Solve the matrix equation for the matrix X: $AX + 3\begin{bmatrix}2\\0\end{bmatrix} = \begin{bmatrix}1\\a\end{bmatrix}$ if $A^{-1} = \begin{bmatrix}4 & 3\\2 & 1\end{bmatrix}$

Note : Complete all operations. The parameter a will be in your answer.

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$AX + 3\begin{bmatrix}2\\0\end{bmatrix} = \begin{bmatrix}1\\a\end{bmatrix}$

$AX + \begin{bmatrix}6\\0\end{bmatrix} = \begin{bmatrix}1\\a\end{bmatrix}$

$AX = \begin{bmatrix}1\\a\end{bmatrix} - \begin{bmatrix}6\\0\end{bmatrix}$

$AX = \begin{bmatrix}-5\\a\end{bmatrix}$

$A^{-1}AX = \begin{bmatrix}4 & 3\\2 & 1\end{bmatrix}\begin{bmatrix}-5\\a\end{bmatrix}$

$\boxed{X = \begin{bmatrix}-20+3a\\-10+a\end{bmatrix}}$

C. (Circle) Which is the matrix A ?

1. $\begin{bmatrix}-1 & 2\\3 & -4\end{bmatrix}$ 2. $\begin{bmatrix}-\frac{1}{2} & \frac{3}{2}\\1 & -2\end{bmatrix}$ 3. $\begin{bmatrix}\frac{1}{4} & -2\\-3 & 1\end{bmatrix}$ 4. $\begin{bmatrix}1 & -2\\-3 & 4\end{bmatrix}$

① Enter A^{-1} into your calculator, storing in matrix B. then $A = [B]^{-1}$ ie $[A^{-1}]^{-1} = A$
2. Multiply possible answers & A^{-1}. you should get I.

Q4: **Leontief Input – Output Analysis:** This is a set-up problem only. You do NOT need to solve.

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An economy is based on three sectors, coal (C), oil (O) and transportation (T).

- Production of \$1 of coal requires inputs from each sector: \$0.15 from C and \$0.40 from both O & T
- Production of \$1 of oil requires inputs from only two sectors: \$0.20 from O and \$0.30 from T.
- Production of \$1 of transportation requires inputs from each sector: \$0.10 from C & O and \$0.35 from T.

Find the total output for each sector that satisfies a final demand of \$30 billion for C, \$20 billion for O and \$40 billion for T.

A. What does the variable x_1 represent? $x_1 =$ total output of Coal to satisfy both internal and external demands.

B. The matrix equation for the set-up is $X = MX + D$ and the solution is $X = (I-M)^{-1}D$
List the elements of each of the matrices.

$X = \begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$ $M = \begin{array}{c c} & \begin{array}{ccc} C & O & T \end{array} \\ \begin{array}{c} C\\O\\T \end{array} & \begin{bmatrix}.15 & 0 & .1\\.4 & .2 & .1\\.4 & .3 & .35\end{bmatrix} \end{array}$ $D = \begin{bmatrix}30\\20\\40\end{bmatrix}$

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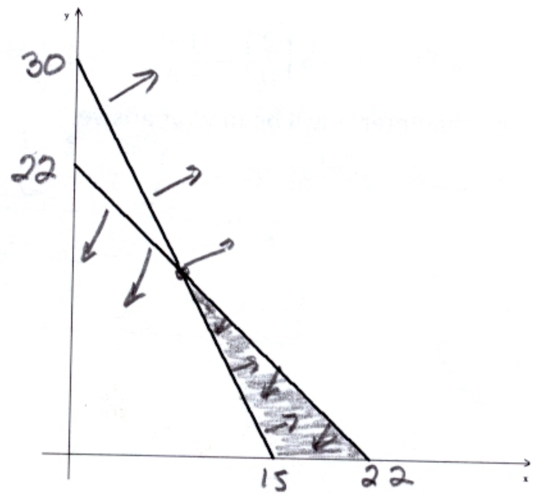
$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} - \begin{bmatrix}.15 & .0 & .1\\.4 & .2 & .1\\.4 & .3 & .35\end{bmatrix}$ $I - M = \begin{bmatrix}.85 & 0 & -.1\\-.4 & .8 & -.1\\-.4 & -.3 & .65\end{bmatrix}$

Q5: Find the maximum value of $P = 2x_1 + 4x_2$ subject to the constraints: $\begin{cases} 2x_1 + x_2 \geq 30 & (0,30)\ (15,0) \\ x_1 + x_2 \leq 22 & (0,22)\ (22,0) \\ x_1, x_2 \geq 0 \end{cases}$



A. The lines are sketched. Shade in the solution region and list the corner points.

Corner pts: $(15,0)$ $(22,0)$ $(8,14)$ see below

$$\begin{array}{r} 2x_1 + x_2 = 30 \\ -(x_1 + x_2 = 22) \\ \hline x_1 + 0 = 8 \end{array} \Rightarrow \begin{array}{l} x_1 + x_2 = 22 \\ 8 + x_2 = 22 \\ x_2 = 14 \end{array}$$

$(x_1, x_2) = (8, 14)$

B. Find the maximum value of the objective function on the feasible region. Show all work.

	$P = 2x_1 + 4x_2$
$(15,0)$	$P = 2(15) + 4(0) = 30$
$(22,0)$	$P = 2(22) + 4(0) = 44$
$(8,14)$	$P = 2(8) + 4(14) = 16 + 56 = 72$

maximum $= 72$ at $(8,14)$

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Q6: An initial simplex tableau for a Standard Maximization Problem with constraints of the form $\leq$ is shown.

8

A. What inequality is associated with the second row of the matrix?

$$\begin{array}{cccccc} x_1 & x_2 & s_1 & s_2 & P & \\ \end{array}$$
$$\left[\begin{array}{ccccc|c} 3 & 4 & 1 & 0 & 0 & 36 \\ 2 & 3 & 0 & 1 & 0 & 20 \\ \hline -15 & -10 & 0 & 0 & 1 & 0 \end{array}\right]$$

$2x_1 + 3x_2 \leq 20$

B. What is the objective function?

$P = 15x_1 + 10x_2$

C. What is the initial basic solution? $x_1 =$ 0 $x_2 =$ 0 $s_1 =$ 36 $s_2 =$ 20 $P =$ 0

D. Circle, on the matrix above, the first pivot element.

$\frac{36}{3} = 12 \quad \frac{20}{2} = 10 \Rightarrow$ 2 is the pivot

Q7: In the simplex tableau below, the current pivot element is circled.

8

A. Record and complete the pivot operations.

$$\begin{array}{cccccc} x_1 & x_2 & s_1 & s_2 & P & \\ \end{array}$$
$$\left[\begin{array}{ccccc|c} 1 & 1 & 1 & 0 & 0 & 30 \\ 2 & 1 & 0 & 1 & 0 & 24 \\ \hline -4 & -2 & 0 & 1 & 1 & 0 \end{array}\right]$$

$\frac{1}{2}R_2 \to R_2$

$$\begin{array}{cccccc} x_1 & x_2 & s_1 & s_2 & P & \\ \end{array}$$
$$\left[\begin{array}{ccccc|c} 1 & 1 & 1 & 0 & 0 & 30 \\ 1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 12 \\ \hline -4 & -2 & 0 & 1 & 1 & 0 \end{array}\right]$$

$-R_2 + R_1 \to R_1$

$4R_2 + R_3 \to R_3$

$$\begin{array}{cccccc} x_1 & x_2 & s_1 & s_2 & P & \\ \end{array}$$
$$\left[\begin{array}{ccccc|c} 0 & \frac{1}{2} & 1 & -\frac{1}{2} & 0 & 18 \\ 1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 12 \\ \hline 0 & 0 & 0 & 2 & 1 & 48 \end{array}\right]$$

B. What is the basic solution? $x_1 =$ 12 $x_2 =$ 0 $s_1 =$ 18 $s_2 =$ 0 $P =$ 48

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Math 1390 – Test #3B

Spring 2010

Name Solution B Row_____

Score: _____ 80

Q1: A trucking firm wants to purchase 10 trucks that will provide exactly 28 tons of additional shipping capacity. A model A truck holds 2 tons, a model B truck holds 3 tons, and a model C truck holds 5 tons. How many trucks of each model should the company purchase to provide the additional shipping capacity?

A. If x_1, x_2 and x_3 represent the number of model A, B and C trucks, respectively, set up the system of linear equations and the augmented matrix that are used to solve this problem.

$$x_1 + x_2 + x_3 = 10$$
$$2x_1 + 3x_2 + 5x_3 = 28$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 10 \\ 2 & 3 & 5 & 28 \end{array}\right]$$

B. When the correct matrix for the problem is row-reduced we get the matrix below. Introduce a parameter t to write the possible solutions of the problem. Recognizing that we are talking about the number of trucks, list the possible values of t.

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 2 \\ 0 & 1 & 3 & 8 \end{array}\right]$$

$2t+2 \geq 0 \Rightarrow 2t \geq -2 \Rightarrow \boxed{t \geq -1}$

$-3t+8 \geq 0 \Rightarrow -3t \geq -8 \Rightarrow \boxed{t \leq \frac{8}{3} \text{ or } 2\frac{2}{3}}$

$\boxed{t \geq 0}$

$\Big\} \; 0 \leq t \leq \frac{8}{3}$

$x_1 =$ $2t+2$

$x_2 =$ $-3t+8$

$x_3 =$ t

where t can equal 0, 1, 2

Since the number of trucks must be a whole #

C. If the purchase prices (in thousands of \$) of the truck models A, B and C are 20, 30, and 50, respectively, what is the total cost of the trucks when $t = 1$?

when $t=1$, $x_1 = 4$, $x_2 = 5$, $x_3 = 1$

$$C = 20(4) + 30(5) + 50(1)$$
$$= 80 + 150 + 50$$
$$= 280$$

or $\boxed{\$\,280{,}000}$

15

Q2: A company makes three chocolate candies: cherry, almond, and raisin. Matrix A gives the number of units of each ingredient, sugar, chocolate and milk, in a batch of each type of candy. Matrix B gives the cost of one unit of each ingredient, in dollars, from suppliers X and Y.

$$A = \begin{array}{r} \\ cherry \\ almond \\ raisin \end{array}\begin{array}{c} \begin{array}{ccc} sugar & choc & milk \end{array} \\ \begin{bmatrix} 4 & 6 & 1 \\ 5 & 3 & 1 \\ 3 & 3 & 1 \end{bmatrix} \end{array} \qquad B = \begin{array}{r} \\ sugar \\ choc \\ milk \end{array}\begin{array}{c} \begin{array}{cc} X & Y \end{array} \\ \begin{bmatrix} 3 & 2 \\ 3 & 4 \\ 2 & 2 \end{bmatrix} \end{array}$$

A. The product of A and B is the matrix $C = \begin{bmatrix} 32 & 34 \\ 26 & 24 \\ 20 & 20 \end{bmatrix}$

(Circle one answer) What does the 24 in this matrix represent?

Using supplier Y:

1. the total cost of sugar in making one batch of each candy
2. the total cost of chocolate in making one batch of each candy
3. the total cost of milk in making one batch of each candy
4. the total cost of all ingredients for making one batch of cherry candy
5. the total cost of all ingredients for making one batch of almond candy
6. the total cost of all ingredients for making one batch of raisin candy

Use the matrices C and $D - G$ to answer the following questions.

$$C = \begin{bmatrix} 32 & 34 \\ 26 & 24 \\ 20 & 20 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 1 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \quad F = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad G = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

3×2 $\quad 1 \times 2$ $\quad 1 \times 3$ $\quad 2 \times 1$ $\quad 3 \times 1$

B. Which two of the following multiplications of the matrix C are legal? Circle your answers.

$CD \quad CE \quad CF \quad CG \quad DC \quad EC \quad FC \quad GC$

C. If we want to use a matrix multiplication to find the total sum of the elements of each column in matrix C, which matrix multiplication that you circled above should be used? answer: EC

10

Q3: A. Circle: Solve for the matrix X in the matrix equation $RX = SX + T$. $X =$

1. $\dfrac{T}{I - RS}$ 2. $T(I - RS)^{-1}$ 3. $(I - RS)^{-1}T$ 4. $(R - S)^{-1}T$ 5. $T(R - S)^{-1}$ 6. $\dfrac{T}{R - S}$

$RX - SX = T \Rightarrow (R - S)X = T \Rightarrow X = (R-S)^{-1}T$

B. Solve the matrix equation for the matrix X: $AX + 2\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ a \end{bmatrix}$ if $A^{-1} = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$

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Note : Complete all operations. The parameter a will be in your answer.

$AX + 2\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ a \end{bmatrix}$

$AX + \begin{bmatrix} 4 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ a \end{bmatrix}$

$AX = \begin{bmatrix} -1 \\ a \end{bmatrix} - \begin{bmatrix} 4 \\ 0 \end{bmatrix} = \begin{bmatrix} -5 \\ a \end{bmatrix}$

$X = \underbrace{\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}}_{2\times 2} \underbrace{\begin{bmatrix} -5 \\ a \end{bmatrix}}_{2\times 1} = \underbrace{\begin{bmatrix} -20+3a \\ -10+5a \end{bmatrix}}_{2\times 1}$

C. (Circle) Which is the matrix A ?

1. $\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}$ 2. $\begin{bmatrix} \frac{1}{4} & -2 \\ -3 & 1 \end{bmatrix}$ 3. $\begin{bmatrix} -\frac{1}{2} & \frac{3}{2} \\ 1 & -2 \end{bmatrix}$ 4. $\begin{bmatrix} -1 & 2 \\ 3 & -4 \end{bmatrix}$

$A = (A^{-1})^{-1}$

Q4: **Leontief Input – Output Analysis:** This is a set-up problem only. You do NOT need to solve.

12

An economy is based on three sectors, coal (C), oil (O) and transportation (T).

- Production of \$1 of coal requires inputs from each sector: \$0.10 from C and \$0.40 from both O & T
- Production of \$1 of oil requires inputs from only two sectors: \$0.20 from O and \$0.35 from T.
- Production of \$1 of transportation requires inputs from each sector: \$0.15 from C & O and \$0.30 from T.

Find the total output for each sector that satisfies a final demand of \$30 billion for C, \$10 billion for O and \$20 billion for T.

A. What does the variable x_1 represent? $x_1 =$ total output of coal needed to satisfy the final demands.

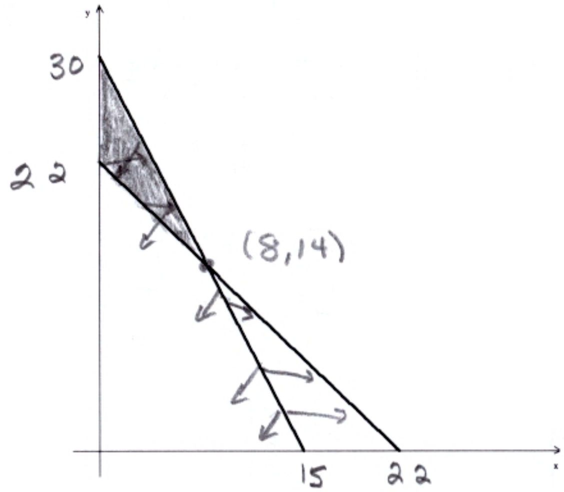
B. The matrix equation for the set-up is $X = MX + D$ and the solution is $X = (I - M)^{-1}D$
List the elements of each of the matrices.

$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ $M = \begin{array}{c} \\ C \\ O \\ T \end{array} \begin{array}{c} \begin{array}{ccc} C & O & T \end{array} \\ \begin{bmatrix} .1 & 0 & .15 \\ .4 & .2 & .15 \\ .4 & .35 & .3 \end{bmatrix} \end{array}$ $D = \begin{bmatrix} 30 \\ 10 \\ 20 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} .1 & 0 & .15 \\ .4 & .2 & .15 \\ .4 & .35 & .3 \end{bmatrix}$ $I - M = \begin{bmatrix} .9 & 0 & -.15 \\ -.4 & .8 & -.15 \\ -.4 & -.35 & .7 \end{bmatrix}$

12

Q5: Find the maximum value of $P = 4x_1 + 2x_2$ subject to the constraints: $\begin{cases} 2x_1 + x_2 \leq 30 & (0,30)\ (15,0) \\ x_1 + x_2 \geq 22 & (0,22)(22,0) \\ x_1, x_2 \geq 0 & \end{cases}$



A. The lines are sketched. Shade in the solution region and list the corner points.

$$\begin{array}{r} 2x_1 + x_2 = 30 \\ -(\ x_1 + x_2 = 22\ ) \\ \hline x_1 + 0 = 8 \end{array}$$

$\boxed{x_1 = 8}$

$8 + x_2 = 22$

$\boxed{x_2 = 14}$

Corner points

(0, 22), (0, 30) (8, 14)

B. Find the maximum value of the objective function on the feasible region. Show all work.

	$P = 4x_1 + 2x_2$
(0,22)	$P = 0 + 2(22) = 44$
(0,30)	$P = 0 + 2(30) = 60$
(8,14)	$P = 4(8) + 2(14) = 32 + 28 = 60$

maximum value of 60 at (0, 30) and (8,14)

and at every point in between.

15

Q6: An initial simplex tableau for a Standard Maximization Problem with constraints of the form $\leq$ is shown.

8

A. What inequality is associated with the second row of the matrix?

$$\begin{array}{cccccc} x_1 & x_2 & s_1 & s_2 & P & \\ \end{array}$$
$$\left[\begin{array}{ccccc|c} 3 & 4 & 1 & 0 & 0 & 36 \\ 2 & 3 & 0 & 1 & 0 & 20 \\ \hline -10 & -15 & 0 & 0 & 1 & 0 \end{array}\right]$$

$2x_1 + 3x_3 \leq 20$

B. What is the objective function?

$P = 10x_1 + 15x_2$

C. What is the initial basic solution? $x_1 =$ 0 $x_2 =$ 0 $s_1 =$ 36 $s_2 =$ 20 $P =$ 0

D. Circle, on the matrix above, the first pivot element.

$\frac{36}{4} = 9 \quad \frac{20}{3} = 6\frac{2}{3} \Rightarrow 3$ is the pivot

Q7: In the simplex tableau below, the current pivot element is circled.

A. Record and complete the pivot operations.

8

$$\begin{array}{cccccc} x_1 & x_2 & s_1 & s_2 & P & \\ \end{array}$$
$$\left[\begin{array}{ccccc|c} 1 & 1 & 1 & 0 & 0 & 30 \\ 2 & 1 & 0 & 1 & 0 & 36 \\ \hline -4 & -2 & 0 & 1 & 1 & 0 \end{array}\right]$$

$\frac{1}{2}R_2 \to R_2$
$$\left[\begin{array}{ccccc|c} 1 & 1 & 1 & 0 & 0 & 30 \\ 1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 18 \\ \hline -4 & -2 & 0 & 1 & 1 & 0 \end{array}\right]$$

$-R_2 + R_1 \to R_1$

$4R_2 + R_3 \to R_3$
$$\left[\begin{array}{ccccc|c} 0 & \frac{1}{2} & 1 & -\frac{1}{2} & 0 & 12 \\ 1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 18 \\ \hline 0 & 0 & 0 & 2 & 1 & 72 \end{array}\right]$$

B. What is the basic solution? $x_1 =$ 18 $x_2 =$ 0 $s_1 =$ 12 $s_2 =$ 0 $P =$ 72

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Math 1390 – Test #3A
Spring 2010

Name ________________________________

Score: _____
80

Q1: A trucking firm wants to purchase 10 trucks that will provide exactly 28 tons of additional shipping capacity. A model A truck holds 2 tons, a model B truck holds 3 tons, and a model C truck holds 5 tons. How many trucks of each model should the company purchase to provide the additional shipping capacity?

A. If x_1, x_2 and x_3 represent the number of model A, B and C trucks, respectively, set up the system of linear equations and the augmented matrix that are used to solve this problem.

B. When the correct matrix for the problem is row-reduced we get the matrix below. Introduce a parameter t to write the possible solutions of the problem. Recognizing that we are talking about the number of trucks, list the possible values of t.

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 2 \\ 0 & 1 & 3 & 8 \end{array}\right]$$

$x_1 =$ __________

$x_2 =$ __________

$x_3 =$ __________ where t can equal ________________________

C. If the purchase prices (in thousands of $) of the truck models A, B and C are 20, 40, and 50, respectively, what is the total cost of the trucks when $t = 1$?

D.

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Q2: A company makes three chocolate candies: cherry, almond, and raisin. Matrix A gives the number of units of each ingredient, sugar, chocolate and milk, in a batch of each type of candy. Matrix B gives the cost of one unit of each ingredient, in dollars, from suppliers X and Y.

$$A = \begin{array}{c} \\ cherry \\ almond \\ raisin \end{array} \begin{array}{c} \begin{array}{ccc} sugar & choc & milk \end{array} \\ \begin{bmatrix} 4 & 6 & 1 \\ 5 & 3 & 1 \\ 3 & 3 & 1 \end{bmatrix} \end{array} \qquad B = \begin{array}{c} \\ sugar \\ choc \\ milk \end{array} \begin{array}{c} \begin{array}{cc} X & Y \end{array} \\ \begin{bmatrix} 3 & 2 \\ 3 & 4 \\ 2 & 2 \end{bmatrix} \end{array}$$

A. The product of A and B is the matrix $C = \begin{bmatrix} 32 & 34 \\ 26 & 24 \\ 20 & 20 \end{bmatrix}$

(Circle one answer) What does the 34 in this matrix represent?

Using supplier Y:

1. the total cost of sugar in making one batch of each candy
2. the total cost of chocolate in making one batch of each candy
3. the total cost of milk in making one batch of each candy
4. the total cost of all ingredients for making one batch of cherry candy
5. the total cost of all ingredients for making one batch of almond candy
6. the total cost of all ingredients for making one batch of raisin candy

Use the matrices C and $D - G$ to answer the following questions.

$$C = \begin{bmatrix} 32 & 34 \\ 26 & 24 \\ 20 & 20 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 1 \end{bmatrix} \qquad E = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \qquad F = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad G = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

B. Which two of the following multiplications of the matrix C are legal? Circle your answers.

$DC \quad EC \quad FC \quad GC \quad CD \quad CE \quad CF \quad CG$

C. If we want to use a matrix multiplication to find the total sum of the elements of each column in matrix C, which matrix multiplication that you circled above should be used? answer: ________

10

Q3: A. Circle: Solve for the matrix X in the matrix equation $RX = SX + T$. $X =$

1. $T(R-S)^{-1}$ 2. $\dfrac{T}{R-S}$ 3. $(R-S)^{-1}T$ 4. $\dfrac{T}{I-RS}$ 5. $T(I-RS)^{-1}$ 6. $(I-RS)^{-1}T$

B. Solve the matrix equation for the matrix X: $AX + 3\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ a \end{bmatrix}$ if $A^{-1} = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$

Note : Complete all operations. The parameter a will be in your answer.

12

$$AX + 3\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ a \end{bmatrix}$$

C. (Circle) Which is the matrix A ?

1. $\begin{bmatrix} -1 & 2 \\ 3 & -4 \end{bmatrix}$ 2. $\begin{bmatrix} -\frac{1}{2} & \frac{3}{2} \\ 1 & -2 \end{bmatrix}$ 3. $\begin{bmatrix} \frac{1}{4} & -2 \\ -3 & 1 \end{bmatrix}$ 4. $\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}$

Q4: **Leontief Input – Output Analysis:** This is a set-up problem only. You do NOT need to solve.

12

An economy is based on three sectors, coal (C), oil (O) and transportation (T).

- Production of $1 of coal requires inputs from each sector: $0.15 from C and $0.40 from both O & T
- Production of $1 of oil requires inputs from only two sectors: $0.20 from O and $0.30 from T.
- Production of $1 of transportation requires inputs from each sector: $0.10 from C & O and $0.35 from T.

Find the total output for each sector that satisfies a final demand of $30 billion for C, $20 billion for O and $40 billion for T.

A. What does the variable x_1 represent? $x_1 =$ ______________________________

B. The matrix equation for the set-up is $X = MX + D$ and the solution is $X = (I-M)^{-1}D$
List the elements of each of the matrices.

$$X = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} \qquad M = \begin{array}{c c} & \begin{array}{ccc} C & O & T \end{array} \\ \begin{array}{c} C \\ O \\ T \end{array} & \begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix} \end{array} \qquad D = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}$$

$$I - M = \begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix}$$

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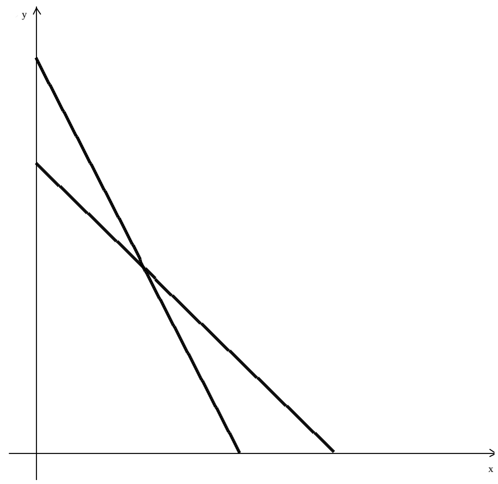
Q5: Find the maximum value of $P = 2x_1 + 4x_2$ subject to the constraints: $\begin{cases} 2x_1 + x_2 \geq 30 \\ x_1 + x_2 \leq 22 \\ x_1, x_2 \geq 0 \end{cases}$



A. The lines are sketched. Shade in the solution region and list the corner points.

B. Find the maximum value of the objective function on the feasible region. Show all work.

15

Q6: An <u>initial</u> simplex tableau for a Standard Maximization Problem with constraints of the form $\leq$ is shown.

8

A. What inequality is associated with the second row of the matrix?

$$\begin{array}{ccccc|c} x_1 & x_2 & s_1 & s_2 & P & \\ \end{array}$$
$$\left[\begin{array}{ccccc|c} 3 & 4 & 1 & 0 & 0 & 36 \\ 2 & 3 & 0 & 1 & 0 & 20 \\ \hline -15 & -10 & 0 & 0 & 1 & 0 \end{array}\right]$$

B. What is the objective function?

C. What is the initial basic solution? $x_1 =$ ______ $x_2 =$ ______ $s_1 =$ ______ $s_2 =$ ______ $P =$ ______

D. Circle, on the matrix above, the first pivot element.

Q7: In the simplex tableau below, the current pivot element is circled.

A. Record and complete the pivot operations.

8

$$\begin{array}{ccccc|c} x_1 & x_2 & s_1 & s_2 & P & \\ \end{array}$$
$$\left[\begin{array}{ccccc|c} 1 & 1 & 1 & 0 & 0 & 30 \\ 2 & 1 & 0 & 1 & 0 & 24 \\ \hline -4 & -2 & 0 & 1 & 1 & 0 \end{array}\right]$$

B. What is the basic solution? $x_1 =$ ______ $x_2 =$ ______ $s_1 =$ ______ $s_2 =$ ______ $P =$ ______

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Math 1390 – Test #3B Score: _____

Spring 2010 Name___________________________________ Row_____ 80

Q1: A trucking firm wants to purchase 10 trucks that will provide exactly 28 tons of additional shipping capacity. A model A truck holds 2 tons, a model B truck holds 3 tons, and a model C truck holds 5 tons. How many trucks of each model should the company purchase to provide the additional shipping capacity?

A. If x_1, x_2 and x_3 represent the number of model A, B and C trucks, respectively, set up the system of linear equations and the augmented matrix that are used to solve this problem.

B. When the correct matrix for the problem is row-reduced we get the matrix below. Introduce a parameter t to write the possible solutions of the problem. Recognizing that we are talking about the number of trucks, list the possible values of t.

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 2 \\ 0 & 1 & 3 & 8 \end{array}\right]$$

$x_1 =$ __________

$x_2 =$ __________

$x_3 =$ __________ where t can equal ________________________

C. If the purchase prices (in thousands of $) of the truck models A, B and C are 20, 30, and 50, respectively, what is the total cost of the trucks when $t = 1$?

D.

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Q2: A company makes three chocolate candies: cherry, almond, and raisin. Matrix A gives the number of units of each ingredient, sugar, chocolate and milk, in a batch of each type of candy. Matrix B gives the cost of one unit of each ingredient, in dollars, from suppliers X and Y.

$$A = \begin{array}{r} \\ cherry \\ almond \\ raisin \end{array} \begin{array}{c} \begin{array}{ccc} sugar & choc & milk \end{array} \\ \begin{bmatrix} 4 & 6 & 1 \\ 5 & 3 & 1 \\ 3 & 3 & 1 \end{bmatrix} \end{array} \qquad B = \begin{array}{r} \\ sugar \\ choc \\ milk \end{array} \begin{array}{c} \begin{array}{cc} X & Y \end{array} \\ \begin{bmatrix} 3 & 2 \\ 3 & 4 \\ 2 & 2 \end{bmatrix} \end{array}$$

A. The product of A and B is the matrix $C = \begin{bmatrix} 32 & 34 \\ 26 & 24 \\ 20 & 20 \end{bmatrix}$

(Circle one answer) What does the 24 in this matrix represent?

Using supplier Y:

1. the total cost of sugar in making one batch of each candy
2. the total cost of chocolate in making one batch of each candy
3. the total cost of milk in making one batch of each candy
4. the total cost of all ingredients for making one batch of cherry candy
5. the total cost of all ingredients for making one batch of almond candy
6. the total cost of all ingredients for making one batch of raisin candy

Use the matrices C and $D-G$ to answer the following questions.

$$C = \begin{bmatrix} 32 & 34 \\ 26 & 24 \\ 20 & 20 \end{bmatrix} \qquad D = [1 \quad 1] \qquad E = [1 \quad 1 \quad 1] \qquad F = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad G = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

B. Which two of the following multiplications of the matrix C are legal? Circle your answers.

$CD \qquad CE \qquad CF \qquad CG \qquad DC \qquad EC \qquad FC \qquad GC$

C. If we want to use a matrix multiplication to find the total sum of the elements of each column in matrix C, which matrix multiplication that you circled above should be used? answer: _______

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Q3: A. Circle: Solve for the matrix X in the matrix equation $RX = SX + T$. $X =$

1. $\dfrac{T}{I - RS}$ 2. $T(I - RS)^{-1}$ 3. $(I - RS)^{-1}T$ 4. $(R - S)^{-1}T$ 5. $T(R - S)^{-1}$ 6. $\dfrac{T}{R - S}$

B. Solve the matrix equation for the matrix X: $AX + 2\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ a \end{bmatrix}$ if $A^{-1} = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$

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Note : Complete all operations. The parameter a will be in your answer.

$AX + 2\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ a \end{bmatrix}$

C. (Circle) Which is the matrix A ?

1. $\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}$ 2. $\begin{bmatrix} \frac{1}{4} & -2 \\ -3 & 1 \end{bmatrix}$ 3. $\begin{bmatrix} -\frac{1}{2} & \frac{3}{2} \\ 1 & -2 \end{bmatrix}$ 4. $\begin{bmatrix} -1 & 2 \\ 3 & -4 \end{bmatrix}$

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$I - M = \begin{bmatrix} \quad & \quad & \quad \\ \\ \\ \end{bmatrix}$

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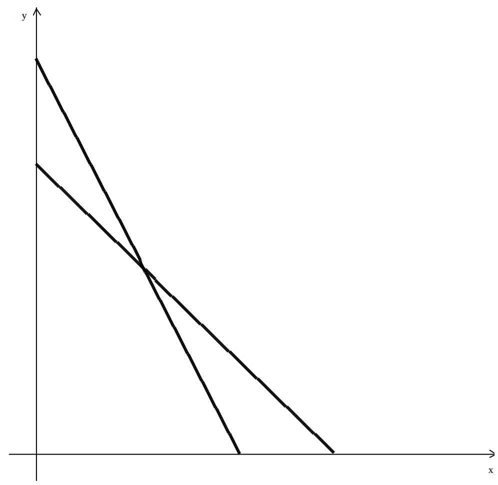

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