6.1 – Geometric Introduction to the Simplex Method

Read pages 292 - 298

Homework: page 297 1, 3, 5, 7

In the Simplex Method, **slack variables** are introduced to convert the constraint inequalities to equalities.

**Definition: Standard Maximazation Problem in Standard Form** *(used in all of 6.1 & 6.2)*

A linear programming problem is said to be a **standard maximization problem in standard form** if its mathematical model is of the following form.

Maximize the objective function \( P = c_1 x_1 + c_2 x_2 + ... + c_n x_n \)

subject to problem contraints of the form \( a_1 x_1 + a_2 x_2 + ... + a_n x_n \leq b, \quad b \geq 0 \)

with non–negative contraints \( x_1, x_2, ..., x_n \geq 0 \)

**Q1:** *Based on #6 page 310* Consider the linear programming problem:

Find the maximum value of \( z = 2x_1 + 3x_2 \), subject to

\[
\begin{align*}
2x_1 + x_2 &\leq 30 \\
x_1 + 5x_2 &\leq 60 \\
x_1, x_2 &\geq 0
\end{align*}
\]

A. Rewrite the constraints as equalities by introducing slack variables.

B. If \( x_1 = 15 \) and \( x_2 = 0 \), what are the values of the slack variables.

**Definitions:** Given a system of linear equations associated with a linear programming problem, the variables are divided into two, mutually exclusive, groups as follows:

- **Basic variables** are selected arbitrarily with the one restriction that the number of basic variables equals the number of equations. The remaining variables are the **nonbasic variables**.
- A **basic solution** is found by setting the nonbasic variables equal to 0 and solving for the basic variables.

If the basic solution has no negative values, it is a **basic feasible solution**.

C. Because we have two equations, \( 2x_1 + x_2 + s_1 = 30 \) and \( x_1 + 5x_2 + s_2 = 60 \), pick two arbitrary variables to be the basic variables and set the remaining variables to be non-basic variables. Find the basic solution if \( x_2 \) and \( s_1 \) are picked to be the basic variables.
Find the maximum value of \( z = 2x_1 + 3x_2 \), subject to
\[
\begin{align*}
2x_1 + x_2 & \leq 30 \\
x_1 + 5x_2 & \leq 60 \\
x_1, x_2 & \geq 0
\end{align*}
\]
\[
\begin{align*}
2x_1 + x_2 + s_1 & = 30 \\
x_1 + 5x_2 + s_2 & = 60
\end{align*}
\]

D. Find all of the basic solutions of the system

<table>
<thead>
<tr>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>Intersection Point</th>
<th>In Feasible Region?</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
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<td></td>
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<tr>
<td>B</td>
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</tr>
<tr>
<td>C</td>
<td>0</td>
<td>12</td>
<td>18</td>
<td>0</td>
<td>(0, 12)</td>
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<tr>
<td>D</td>
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</tr>
<tr>
<td>E</td>
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<td>F</td>
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</tbody>
</table>

E. Find the maximum value of the objective function on the feasible region.
Q2: Discuss the relationship between a standard maximization problem with three problem constraints and four decision variables, and the associated system of problem constraint equations. In particular, find the following quantities and explain how each was determined.

A. The number of slack variables that must be introduced to form the system of problem constraint equations.

B. The number of basic variables and the number of non-basic variables associated with the system.

C. The number of linear equations and the number of variables in the system formed by setting the nonbasic variables equal to 0.