4.4 Matrices: Basic Operations

Homework: page 220 1, 3, 5, 7, 9, 11, 13, 17, 19, 21, 23, 27, 31, 33, 37, 41, 51, 53, 61, 65, 71

Matrix Addition, Subtraction and constant Multiplication

1. Two matrices are equal if they have the same size and their corresponding elements are equal.
2. **Addition:** The sum of two matrices of the same size is the matrix with elements that are the sum of the corresponding elements of the two matrices.

3. Matrix addition is **commutative:** \( A + B = B + A \)
   and **associative:** \( (A + B) + C = A + (B + C) \)
   **Note:** Real number addition is **commutative:** \( a + b = b + a \)
   and **associative:** \( (a + b) + c = a + (b + c) \)

4. The **zero matrix**, sometimes referred to as \([0]\), is any matrix whose elements are all 0.
5. The **opposite** of the matrix \( A \) is the matrix \(-A\), whose elements are the opposites of the corresponding elements in \( A \). \( A + (-A) = 0 \)
   **Note:** The opposite of the real number \( a \) is \(-a\) and \( a + (-a) = 0 \)

6. **Subtraction:** The difference of two matrices of the same size is \( A - B = A + (-B) \)
   **Note:** The difference of two real numbers \( a - b = a + (-b) \)

7. The product of a number \( k \) and a matrix \( M, kM \), is a matrix formed by multiplying each element of \( M \) by \( k \).

Q1: Find the following.

| A. \[
\begin{bmatrix}
-3 & 5 \\
2 & 0 \\
1 & 4
\end{bmatrix}
+ 
\begin{bmatrix}
2 & 1 \\
-6 & 3 \\
0 & -5
\end{bmatrix}
| B. \[
\begin{bmatrix}
-3 & 5 \\
2 & 0 \\
1 & 4
\end{bmatrix}
+ 
\begin{bmatrix}
-2 & 1 \\
5 & 6 \\
6 & -8
\end{bmatrix}
|
| C. 10\[
\begin{bmatrix}
2 & -1 \\
0 & 5
\end{bmatrix}
| D. \[
\begin{bmatrix}
6 & 2 \\
0 & -4
\end{bmatrix}
- 
\begin{bmatrix}
3 & -1 \\
-2 & 5
\end{bmatrix}
|
Q2: Find $w, x, y$ and $z$ so that

\[
\begin{bmatrix}
4 & -2 \\
-3 & 0
\end{bmatrix} + \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -3 \\ 5 \end{bmatrix}
\]

Q3: If possible, perform the multiplications:

A. \[
\begin{bmatrix}
1 & -2 & 3
\end{bmatrix} \begin{bmatrix}
2 \\
-1 \\
3
\end{bmatrix}
\]

B. \[
\begin{bmatrix}
-4 & 3
\end{bmatrix} \begin{bmatrix}
-2 \\
1
\end{bmatrix}
\]

C. \[
\begin{bmatrix}
-2 \\
1
\end{bmatrix} \begin{bmatrix}
-4 & 3
\end{bmatrix}
\]

---

### Matrix Multiplication

1. The product of a $1 \times n$ row matrix and a $n \times 1$ column matrix is a $1 \times 1$ matrix given by

\[
\begin{bmatrix}
a_1 & a_2 & \ldots & a_n
\end{bmatrix}
\begin{bmatrix}
b_1 \\
b_2 \\
\vdots \\
b_n
\end{bmatrix} = \begin{bmatrix}
a_1b_1 + a_2b_2 + \ldots + a_nb_n
\end{bmatrix}
\]

**Note:** It is common for the $1 \times 1$ product matrix to be written as a real number omitting the brackets.

2. If $A$ is an $m \times p$ matrix and $B$ is a $p \times n$ matrix, the matrix product of $A$ and $B$, denoted $AB$, is an $m \times n$ matrix whose element in the $i^{th}$ row and $j^{th}$ column $B$ is the real number obtained by multiplying the $i^{th}$ row of $A$ and the $j^{th}$ column of $B$. If the number of columns in $A$ does not equal the number of rows in $B$, the matrix product $AB$ is not defined.

**Note 1:** Matrix multiplication is not commutative, i.e. $AB$ does not always equal $BA$.

This is different than real number multiplication, which is commutative.

**Note 2:** If $AB$ is the zero matrix, it is possible that neither $A$ nor $B$ is a zero matrix.

This is different than real number multiplication, where the statement $ab = 0$ implies that at least $a = 0$ or $b = 0$ or both equal 0.
D. \[ \begin{bmatrix} 1 & 2 & 4 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 0 & -2 \\ 1 & 3 \end{bmatrix} \]

F. \[ \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} -2 & 4 \\ 1 & -2 \end{bmatrix} \]

G. \[ \begin{bmatrix} 1 & 2 & 4 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix} \]

Q4: Find \( a, b, c \) and \( d \).

\[ \begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 6 & -5 \\ 7 & -7 \end{bmatrix} \]
Q5: A personal computer retail company sells five different computer models through three stores in a city. The inventory of each model on hand in each store is summarized in matrix A. Wholesale (W) and retail (R) values of each model of computer are summarized in matrix B.

\[
A = \begin{bmatrix}
4 & 2 & 3 & 7 & 1 \\
2 & 3 & 5 & 0 & 6 \\
10 & 4 & 3 & 4 & 3 \\
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
700 & 1400 \\
840 & 1800 \\
1400 & 1800 \\
2700 & 2400 \\
3300 & 3500 \\
4900 & 3500 \\
\end{bmatrix}
\]

A. What is the retail value of the inventory at store 2?

B. What is the wholesale value of the inventory at store 3?

C. If either of the products AB or BA has a meaningful interpretation, find the product and label the rows and columns.

E. Discuss methods of matrix multiplication that can be used to find the total inventory of each model on hand in all three stores. State the matrices that can be used and perform the necessary operations.

F. Discuss methods of matrix multiplication that can be used to find the total inventory of all five models at each store. State the matrices that can be used and perform the necessary operations.