4.1 – Systems of Linear Equations

Homework problem # 53 on page 187

At $4.80 per bushel, the annual supply for soybeans is 1.9 billion bushels and the annual demand is 2.0 billion bushels. When the price increases to $5.10 per bushel, the annual supply increases to 2.1 billion bushels and the annual demand decreases to 1.8 billion bushels. Assume that the supply and demand equations are linear.

Find the supply and demand equations .

Let $p =$ price in dollars
$q =$ quantity of soybeans in billions of bushels.

Organize the given information into a chart.

<table>
<thead>
<tr>
<th>price p</th>
<th>Supply - q</th>
<th>Demand - q</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.80</td>
<td>1.9</td>
<td>2.0</td>
</tr>
<tr>
<td>5.10</td>
<td>2.1</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Therefore, the supply equation passes through the points $(q, p)$: $(1.9, 4.80)$ & $(2.1, 5.10)$

and has slope $\frac{5.10 - 4.80}{2.1 - 1.9} = \frac{0.3}{0.2} = 1.5$

and form: $p = 1.5q + b$.

Substituting the first point into the equation and solving for $b$, we get

$4.80 = 1.5(1.9) + b$
$4.80 = 2.85 + b$
$b = 1.95$
$p = 1.5q + 1.95$

Similarly, the demand equation has slope $\frac{5.10 - 4.80}{1.8 - 2.0} = \frac{0.3}{-0.2} = -1.5$.

Substituting the first point into the equation:

$4.80 = -1.5(2.0) + b$
$4.80 = -3 + b$
$b = 7.8$
$p = -1.5q + 7.8$

To find the equilibrium point, solve the system of equations

\[
\begin{align*}
  p &= 1.5q + 1.95 \\
  p &= -1.5q + 7.8 \\
\text{Add} \quad 2p &= 9.75 \\
  p &= 4.875
\end{align*}
\]

Then solve for $q$: $4.875 = 1.5q + 1.95$
$1.5q = 4.875 - 1.95$
$q = \frac{2.925}{1.5} = 1.95$

Equilibrium point is at a price of $4.875 dollars per bushel and a quantity of 1.95 billion bushels of soybeans.