Exam 2 — Extra Practice Solutions

1. Suppose that a medical parts supplier produces parts with a mean length of 1 cm. Assume that the part lengths are normally distributed. How small should the standard deviation be to guarantee that at least 97% of the parts have a diameter between 0.98 and 1.02 cm?

   Solution: Let x be the diameter of the part. It is normal with $\mu = 1$, and we want $P(0.98 < x < 1.02) = .97$. Since we're looking for the same distance above and below the mean, we first look for an a with $P(-a < z < a) = .97$. By drawing a picture and noting that the complementary probability of .03 must be split up evenly among both the $z > a$ and $z < -a$ parts, we see that we want $P(z < a) = .97 + .015 = .985$. The table gives $a = 2.17$. Now, using the standardization

   $$z = \frac{x - \mu}{\sigma}$$

   and noting that $a = 2.17$ is the z-value we want (on the upper end of the interval),

   $$2.17 = \frac{1.02 - 1}{\sigma} = \frac{.02}{\sigma}$$

   or $\sigma = .02/2.17 = .0092$. Note: we could just as easily have used the $-a = -2.17$ value and calculated

   $$-2.17 = \frac{.98 - 1}{\sigma};$$

   we would get the same value for σ.

2. When functioning normally, a machine produces delicious candies with the average weight (of a single candy) of 0.5 ounces. Whether functioning normally or not, the standard deviation in the weight of a single candy is 0.02. The candies are then packaged 40 to a box, for a theoretical net weight of 20 ounces.

   (a) What is the distribution of the TOTAL weight of candies in a box?

   (b) If the machine is functioning normally, what is the probability that the TOTAL weight in a box is less than 19.8 ounces?

   (c) Suppose that Samuel has messed with the calibration dials, and the mean weight of each candy is actually 0.497 ounces. Now what is the probability that the total weight of a box is less than 19.8 ounces?

3. **Solution:**

   - (a) The average weight of a candy is $\mu = .5$ when the machine functions normally, and the st dev is always 0.02. Since $40 > 30$, we consider the 40 candies in the box a large enough sample, so the Central Limit Theorem applies and says that the sum of the candies (call it w) is approximately a normal random variable with mean $n\mu = 40(.5) = 20$ and standard deviation $\sigma\sqrt{n} = 0.02\sqrt{40} \approx 0.126$.

- (b)

$$P(w < 19.8) = P(z < \frac{19.8 - 20}{0.126}) = P(z < -1.587) = .0561.$$

- (c) Now we have that $\mu = .497$, so w has mean $n\mu = 19.88$ and standard deviation $\sigma\sqrt{n} = 0.126$ (this hasn't changed). This time we get

$$P(w < 19.8) = P(z < \frac{19.8 - 19.88}{0.126}) = P(z < -.635) = .2627.$$