

$$\frac{[y_2 - (y_1 - \mu_1)]^2}{\sigma_1^2} + \frac{(y_2 - \mu_2)^2}{\sigma_2^2} = \frac{y_2^2 - 2(y_1 - \mu_1)y_2 + (y_1 - \mu_1)^2}{\sigma_1^2} + \frac{y_2^2 - 2\mu_2 y_2 + \mu_2^2}{\sigma_2^2}$$

$$= y_2^2 \left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2} \right) - 2y_2 \left(\frac{(y_1 - \mu_1)}{\sigma_1^2} + \frac{\mu_2}{\sigma_2^2} \right) + \frac{(y_1 - \mu_1)^2}{\sigma_1^2} + \frac{\mu_2^2}{\sigma_2^2}$$

$$= \left(\frac{\sigma_1^2 + \sigma_2^2}{\sigma_1^2 \sigma_2^2} \right) \left[y_2^2 - 2y_2 \underbrace{\left(\frac{\sigma_2^2 (y_1 - \mu_1) + \sigma_1^2 \mu_2}{\sigma_1^2 \sigma_2^2} \right)}_{\delta} \right] + \underbrace{\frac{(y_1 - \mu_1)^2}{\sigma_1^2} + \frac{\mu_2^2}{\sigma_2^2}}_{\gamma}$$

$$= \tau^{-2} \left[y_2 - \delta \right]^2 - \tau^{-2} \delta^2 + \gamma$$

$$\gamma = \frac{(y_1 - \mu_1)^2 \sigma_2^2 + \mu_2^2 \sigma_1^2}{\sigma_1^2 \sigma_2^2}$$

$$\delta = \frac{\sigma_2^2 (y_1 - \mu_1) + \sigma_1^2 \mu_2}{\sigma_1^2 + \sigma_2^2}$$

$$- \tau^{-2} \delta^2 + \gamma = \frac{-\sigma_1^2 + \sigma_2^2}{\sigma_1^2 \sigma_2^2} \left[\frac{\sigma_2^2 (y_1 - \mu_1) + \sigma_1^2 \mu_2}{\sigma_1^2 + \sigma_2^2} \right]^2 + \frac{(y_1 - \mu_1)^2 \sigma_2^2 + \mu_2^2 \sigma_1^2}{\sigma_1^2 \sigma_2^2}$$

$$= \frac{-1}{\sigma_1^2 \sigma_2^2} \frac{\sigma_2^4 (y_1 - \mu_1)^2 + 2\sigma_2^2 (y_1 - \mu_1) \sigma_1^2 \mu_2 + \sigma_1^4 \mu_2^2}{\sigma_1^2 + \sigma_2^2}$$

$$+ \frac{(y_1 - \mu_1)^2 \sigma_2^2 + \mu_2^2 \sigma_1^2}{\sigma_1^2 \sigma_2^2}$$

$$= \frac{1}{\sigma_1^2} (y_1^2 - 2\mu_1 y_1 + \mu_1^2) + \frac{\mu_2^2}{\sigma_2^2}$$

$$- \frac{1}{\sigma_1^2 \sigma_2^2} \frac{\sigma_2^4 (y_1^2 - 2\mu_1 y_1 + \mu_1^2) + 2\sigma_2^2 \sigma_1^2 \mu_2 y_1 - 2\sigma_2^2 \sigma_1^2 \mu_1 \mu_2 + \sigma_1^4 \mu_2^2}{\sigma_1^2 + \sigma_2^2}$$

$$= y_1^2 \left[\frac{1}{\sigma_1^2} - \frac{\sigma_2^4}{\sigma_1^2 \sigma_2^2 (\sigma_1^2 + \sigma_2^2)} \right] - 2 \left[\frac{\mu_1}{\sigma_1^2} - \frac{\sigma_2^2 \mu_1^2}{\sigma_1^2 (\sigma_1^2 + \sigma_2^2)} - \frac{\mu_2}{(\sigma_1^2 + \sigma_2^2)} \right] y_1$$

$$+ \text{stuff} = \frac{(\sigma_1^2 + \sigma_2^2) - \sigma_2^2}{\sigma_1^2 (\sigma_1^2 + \sigma_2^2)} y_1^2 - 2 \left[\frac{\mu_1 (\sigma_1^2 + \sigma_2^2) - \sigma_2^2 \mu_1 + \sigma_1^2 \mu_2}{\sigma_1^2 (\sigma_1^2 + \sigma_2^2)} \right] y_1$$

$$= \frac{1}{(\sigma_1^2 + \sigma_2^2)} \left[y_1^2 - 2(\mu_1 + \mu_2) y_1 \right] + \text{stuff}$$

$$= \frac{1}{(\sigma_1^2 + \sigma_2^2)} \left[y_1 - (\mu_1 + \mu_2) \right]^2$$