Class 13

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Agenda:

Recap Chapter 6.3 – 6.5

Lecture Chapter 7.1 – 7.2

Review Exam 3 Material.

Problem Solving Session.
Recap Chapter 6.3 - 6.5
Example:
Assume that IQ scores are normally distributed with a mean \( \mu \) of 100 and a standard deviation \( \sigma \) of 16.

If a person is picked at random, what is the probability that his or her IQ is between 100 and 115?

i.e. \( P(100 < x < 115) \)?

Figures from Johnson & Kuby, 2012.
6: Normal Probability Distributions
6.3 Applications of Normal Distributions

IQ scores normally distributed
\( \mu = 100 \) and \( \sigma = 16 \).

\[ P(100 < x < 115) \]

\[ z = \frac{x - \mu}{\sigma} \]

\[ z_1 = \frac{x_1 - \mu}{\sigma} = \frac{100 - 100}{16} = 0 \]

\[ z_2 = \frac{x_2 - \mu}{\sigma} = \frac{115 - 100}{16} = 0.94 \]

Figures from Johnson & Kuby, 2012.
Now we can use the table.

\[ P(0 < z < 0.94) = P(z < 0.94) - P(z < 0) \]
\[ = 0.8264 - 0.5 \]
\[ = 0.3264 \]

Figures from Johnson & Kuby, 2012.
6: Normal Probability Distributions

6.4 Notation

Example:
Let $\alpha=0.05$. Let’s find $z(0.05)$.

$P(z > z(0.05)) = 0.05$.

Same as finding $P(z < z(0.05)) = 1 - 0.05$.

Figures from Johnson & Kuby, 2012.
6: Normal Probability Distributions

6.4 Notation

Example:
Same as finding $P(z < z(0.05)) = 0.95$.

Figures from Johnson & Kuby, 2012.
Approximate binomial probabilities with normal areas.

Use a normal with \( \mu = np, \ \sigma^2 = np(1 - p) \).

\[
\mu = (14)(.5) = 7
\]

\[
\sigma^2 = (14)(.5)(1 - .5) = 3.5
\]

Figures from Johnson & Kuby, 2012.
6: Normal Probability Distributions
6.5 Normal Approximation of the Binomial Distribution

We then approximate binomial probabilities with normal areas.

\[ P(x = 4) \] from the binomial formula

is approximately \( P(3.5 < x < 4.5) \)

from the normal with \( \mu = 7, \sigma^2 = 3.5 \)

the \( \pm 0.5 \) is called a “continuity correction”

Figures from Johnson & Kuby, 2012.
6: Normal Probability Distributions

6.5 Normal Approximation of the Binomial Distribution

From the binomial formula

\[ P(4) = \frac{14!}{4!(14-4)!} \cdot (0.5)^4 (1-0.5)^{14-4} \]

\[ P(x = 4) = 0.061 \]

\[ P(-1.87 < z < -1.34) = 0.0594 \]

From the Normal Distribution

\[ P(3.5 < x < 4.5) \quad \mu = 7, \quad \sigma^2 = 3.5 \]

\[ \sigma = 1.87 \]

\[ z_1 = \frac{x_1 - \mu}{\sigma} = \frac{3.5 - 7}{1.87} = -1.87 \]

\[ z_2 = \frac{x_2 - \mu}{\sigma} = \frac{4.5 - 7}{1.87} = -1.34 \]

\[ P(z < -1.87) \]

\[ P(z < -1.34) \]

\[ P(z < -1.34) = 0.0901 \]

\[ P(z < -1.34) = 0.0307 \]

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6: Normal Probability Distributions

Questions?

Homework: Chapter 6 # 7, 9, 13, 17, 19, 29, 31, 33, 41, 45, 47, 53, 61, 75, 95, 99
Read Chapter 7.
Lecture Chapter 7.1- 7.2
Chapter 7: Sample Variability

Daniel B. Rowe, Ph.D.

Department of Mathematics, Statistics, and Computer Science
7: Sample Variability
7.2 The Sampling Distribution of Sample Means

When we take a random sample \(x_1, \ldots, x_n\) from a population, one of the things that we do is compute the sample mean \(\bar{x}\).

The value of \(\bar{x}\) is not \(\mu\). Each time we take a random sample of size \(n\), we get a different set of values \(x_1, \ldots, x_n\) and a different value for \(\bar{x}\).
7: Sample Variability

7.2 The Sampling Distribution of Sample Means

**Recall:** When we take a sample of data \( x_1, \ldots, x_n \) from a population, then compute an estimate of a parameter it is called a sample statistic. i.e. \( \bar{x} \) for \( \mu \)

**Sampling Distribution of a sample statistic:** The distribution of values for a sample statistic obtained from repeated samples, all of the same size and all drawn from the same population.
Let’s discuss the relationship between the sample mean and the population mean.

Assume that we have a population of items with population mean $\mu$ and population standard deviation $\sigma$.

If we take a random sample of size $n$ and compute sample mean, $\bar{x}$.

The collection of all possible means is called the sampling distribution of the sample mean.
7: Sample Variability

7.2 The Sampling Distribution of Sample Means

Example:

$N=5$ balls in bucket, select $n=1$ with replacement.

$S=\{\}$

Prob. of each value =
7: Sample Variability
7.2 The Sampling Distribution of Sample Means

Example:
$N=5$ balls in bucket, select $n=1$ with replacement.
Population data values:
0, 2, 4, 6, 8.

$S=\{0, 2, 4, 6, 8\}$

$x = 0$, occurs one time
$x = 2$, occurs one time
$x = 4$, occurs one time
$x = 6$, occurs one time
$x = 8$, occurs one time

Prob. of each value $= 1/5 = 0.2$
7: Sample Variability
7.2 The Sampling Distribution of Sample Means

Example:
\( N = 5 \) balls in bucket, select \( n = 1 \) \textit{with} replacement.
Population data values: 0, 2, 4, 6, 8.

\[
\begin{array}{c|c}
 x & P(x) \\
 0 & 1/5 \\
 2 & 1/5 \\
 4 & 1/5 \\
 6 & 1/5 \\
 8 & 1/5 \\
\end{array}
\]
7: Sample Variability
7.2 The Sampling Distribution of Sample Means

Example:
N=5 balls in bucket, select n=1 with replacement.
Population data values: 0, 2, 4, 6, 8. 5 possible values

\[
\begin{array}{c|c}
  x & P(x) \\
  \hline
  0 & 1/5 \\
  2 & 1/5 \\
  4 & 1/5 \\
  6 & 1/5 \\
  8 & 1/5 \\
\end{array}
\]

\[
\mu = \sum [xP(x)] =
\]
7: Sample Variability
7.2 The Sampling Distribution of Sample Means

Example:
$N=5$ balls in bucket, select $n=1$ with replacement.
Population data values: 0, 2, 4, 6, 8.

5 possible values

\[
\begin{array}{c|c}
 x & P(x) \\
 0 & 1/5 \\
 2 & 1/5 \\
 4 & 1/5 \\
 6 & 1/5 \\
 8 & 1/5 \\
\end{array}
\]

\[
\sigma^2 = \sum [(x - \mu)^2 P(x)]
\]

= 

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7: Sample Variability
7.2 The Sampling Distribution of Sample Means

Example:
$N=5$ balls in bucket, select $n=2$ with replacement.

Population data values:
0, 2, 4, 6, 8.

25 possible samples
7: Sample Variability
7.2 The Sampling Distribution of Sample Means

Example: There are \( N=5 \) items in the population. Population data values: 0, 2, 4, 6, 8. Take samples of size \( n=2 \) (with replacement).

There are 25 possible samples.

<table>
<thead>
<tr>
<th>Sample 1</th>
<th>Sample 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>(0,0)</td>
</tr>
<tr>
<td>(0,2)</td>
<td>(0,2)</td>
</tr>
<tr>
<td>(0,4)</td>
<td>(0,4)</td>
</tr>
<tr>
<td>(0,6)</td>
<td>(0,6)</td>
</tr>
<tr>
<td>(0,8)</td>
<td>(0,8)</td>
</tr>
<tr>
<td>(2,0)</td>
<td>(2,0)</td>
</tr>
<tr>
<td>(2,2)</td>
<td>(2,2)</td>
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<tr>
<td>(2,4)</td>
<td>(2,4)</td>
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<td>(2,6)</td>
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<td>(2,8)</td>
<td>(2,8)</td>
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<tr>
<td>(4,0)</td>
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<tr>
<td>(4,2)</td>
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<tr>
<td>(4,4)</td>
<td>(4,4)</td>
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<tr>
<td>(4,6)</td>
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<td>(4,8)</td>
<td>(4,8)</td>
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<tr>
<td>(6,0)</td>
<td>(6,0)</td>
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<tr>
<td>(6,2)</td>
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<td>(6,4)</td>
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<td>(6,6)</td>
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<td>(6,8)</td>
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<tr>
<td>(8,0)</td>
<td>(8,0)</td>
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<tr>
<td>(8,2)</td>
<td>(8,2)</td>
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<td>(8,4)</td>
<td>(8,4)</td>
</tr>
<tr>
<td>(8,6)</td>
<td>(8,6)</td>
</tr>
<tr>
<td>(8,8)</td>
<td>(8,8)</td>
</tr>
</tbody>
</table>

Each sample has mean \( \bar{x} \).
7: Sample Variability
7.2 The Sampling Distribution of Sample Means

**Example:** \(N=5\), values: 0, 2, 4, 6, 8, \(n=2\) (with replacement).
25 possible samples.

Each possible sample is equally likely.

Prob. of each sample

\[
P[(i, j)] = \frac{1}{25}
\]

\(i = 0, 2, 4, 6, 8\)

\(j = 0, 2, 4, 6, 8\)
7: Sample Variability

7.2 The Sampling Distribution of Sample Means

Example: \( N=5 \), values: 0, 2, 4, 6, 8, \( n=2 \) (with replacement).
25 possible samples. Each possible sample is equally likely.

Prob. of each sample mean = \( 1/25 = 0.04 \)

\[ \begin{align*}
\text{There are 25 possible samples.} \\
(0,0) & \quad (2,0) & \quad (4,0) & \quad (6,0) & \quad (8,0) \\
(0,2) & \quad (2,2) & \quad (4,2) & \quad (6,2) & \quad (8,2) \\
(0,4) & \quad (2,4) & \quad (4,4) & \quad (6,4) & \quad (8,4) \\
(0,6) & \quad (2,6) & \quad (4,6) & \quad (6,6) & \quad (8,6) \\
(0,8) & \quad (2,8) & \quad (4,8) & \quad (6,8) & \quad (8,8)
\end{align*} \]
7: Sample Variability
7.2 The Sampling Distribution of Sample Means

Example: \( N=5 \), values: 0, 2, 4, 6, 8, \( n=2 \) (with replacement). 25 possible samples.

\[ \bar{x} = ?, \text{ occurs } xxx \text{ times} \]
\[ \bar{x} = ?, \text{ occurs } xxx \text{ times} \]
\[ \bar{x} = ?, \text{ occurs } xxxxxx \text{ times} \]
\[ \bar{x} = ?, \text{ occurs } xx \text{ times} \]
\[ \bar{x} = ?, \text{ occurs } xxxxx \text{ times} \]
\[ \bar{x} = ?, \text{ occurs } xxxx \text{ times} \]
\[ \bar{x} = ?, \text{ occurs } xxx \text{ times} \]
\[ \bar{x} = ?, \text{ occurs } xxx \text{ times} \]

Prob. of each samples mean = \( \frac{1}{25} = 0.04 \)}
7: Sample Variability

7.2 The Sampling Distribution of Sample Means

Example: \( N=5 \), values: 0, 2, 4, 6, 8, \( n=2 \) (with replacement).

25 possible samples.

Prob. of each samples mean = \( 1/25 = 0.04 \)

\[
P(\bar{x} = ?) = \]
\[
P(\bar{x} = ?) = \]
\[
P(\bar{x} = ?) = \]
\[
P(\bar{x} = ?) = \]
\[
P(\bar{x} = ?) = \]
\[
P(\bar{x} = ?) = \]
\[
P(\bar{x} = ?) = \]
\[
P(\bar{x} = ?) = \]
7: Sample Variability
7.2 The Sampling Distribution of Sample Means

Don’t forget that the two values that we draw are random.

That is, we may know the sample space of possible outcomes but we do not know exactly which ones we will get!

**Random Sample:** A sample obtained in such a way that each possible sample of fixed size $n$ has an equal probability of being selected.
7: Sample Variability

7.2 The Sampling Distribution of Sample Means

As the number of samples increases the empirical dist. turns into theoretical dist.

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Sample distribution of sample means (SDSM): If all possible random samples, each of size $n$, are taken from any population with mean $\mu$ and standard deviation $\sigma$, then the sampling distribution of sample means will have the following:

1. A mean $\mu_{\bar{x}}$ equal to $\mu$
2. A standard deviation $\sigma_{\bar{x}}$ equal to $\frac{\sigma}{\sqrt{n}}$

Furthermore, if the sampled population has a normal distribution, then the sampling distribution of $\bar{x}$ will also be normal for all samples of all sizes.

Discuss Later: What if the sampled population does not have a normal distribution?
7: Sample Variability

7.2 The Sampling Distribution of Sample Means

As the number of samples increases the empirical dist. turns into theoretical dist.

true distribution with population parameters

\[ \mu, \sigma \]

parameter of interest, \( \mu \)

empirical distribution

\[ \text{portion of Figure from Johnson & Kuby, 2012.} \]

\[ \mu \overline{x} = \mu \text{ and } \sigma \overline{x} = \frac{\sigma}{\sqrt{n}} \]
7: Sample Variability
7.2 The Sampling Distribution of Sample Means

Example: \( N=5 \), values: 0, 2, 4, 6, 8, \( n=2 \) (with replacement).
Instead of drawing two values with replacement and computing the sample mean, we can think of this as drawing one of the sample means with replacement.

The probability for each sample mean is →
7: Sample Variability
7.2 The Sampling Distribution of Sample Means

Example: \( N=5 \), values: 0, 2, 4, 6, 8, \( n=2 \) (with replacement).

\[
\mu_\bar{x} = \sum \bar{x} P(\bar{x})
\]

\[
\mu_\bar{x} = \text{?}
\]

\[
P(\bar{x} = ?) = \text{?}
\]

\[
P(\bar{x} = ?) = \text{?}
\]

\[
P(\bar{x} = ?) = \text{?}
\]

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P(\bar{x} = ?) = \text{?}
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P(\bar{x} = ?) = \text{?}
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P(\bar{x} = ?) = \text{?}
\]

\[
P(\bar{x} = ?) = \text{?}
\]

\[
P(\bar{x} = ?) = \text{?}
\]
7: Sample Variability
7.2 The Sampling Distribution of Sample Means

Example: \( N=5 \), values: 0, 2, 4, 6, 8, \( n=2 \) (with replacement).

\[
\sigma_x^2 = \sum (\overline{x} - \mu)^2 P(\overline{x})
\]

\[
\sigma_x^2 =
\]

\[
P(\overline{x} = ?) = 
\]

\[
P(\overline{x} = ?) = 
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P(\overline{x} = ?) = 
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P(\overline{x} = ?) = 
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P(\overline{x} = ?) = 
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P(\overline{x} = ?) = 
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P(\overline{x} = ?) = 
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P(\overline{x} = ?) = 
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\[
P(\overline{x} = ?) = 
\]
7: Sample Variability

Questions?

Homework: Chapter 7 # 6, 21, 23, 29, 33, 35
Review Chapters 5
(Exam 3 Chapter)

Just the highlights!
5: Probability Distributions (Discrete Variables)

5.2 Probability Distributions of a Discrete Random Variable

Random Variables: ... assumes a unique ... value for each of the outcomes in the sample space ... .

Probability Function: A rule $P(x)$ that assigns probabilities to the values of the random variable $x$.

Example:
Let $x =$ # of heads when we flip a coin twice.

$x = \{0, 1, 2\}$

$$P(x) = \frac{2!}{x!(2-x)!} \left( \frac{1}{2} \right)^x \left( \frac{1}{2} \right)^{2-x}$$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$P(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>1</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{1}{4}$</td>
</tr>
</tbody>
</table>
Mean of a discrete random variable (expected value):
The mean, $\mu$, of a discrete random variable $x$ is found by multiplying each possible value of $x$ by its own probability, $P(x)$, and then adding all of the products together:

$$\mu = \sum_{i=1}^{n} [x_i P(x_i)]$$  \hspace{1cm} (5.1)
5: Probability Distributions (Discrete Variables)
5.2 Mean and Variance of a Discrete Random Variable

\[ \mu = \sum_{i=1}^{n} [x_i P(x_i)] = x_1 P(x_1) + x_2 P(x_2) + \ldots + x_n P(x_n) \]

For the # of \( H \) when we flip a coin twice discrete distribution:

\[ \mu = (x_1) \cdot P(x_1) + (x_2) \cdot P(x_2) + (x_3) \cdot P(x_3) \]

\[ \mu = (0) \cdot P(0) + (1) \cdot P(1) + (2) \cdot P(2) \]

\[ \mu = (0) \cdot (1/4) + (1) \cdot (1/2) + (2) \cdot (1/4) \]

\[ \mu = 0 + 1/2 + 1/2 \]

\[ \mu = 1 \]
5: Probability Distributions (Discrete Variables)
5.2 Mean and Variance of a Discrete Random Variable

Variance of a discrete random variable: The variance, $\sigma^2$, of a discrete random variable $x$ is found by multiplying each possible value of the squared deviation, $(x - \mu)^2$, by its own probability, $P(x)$, and then adding all of the products together:

variance of $x$: sigma squared

$$\sigma^2 = \sum_{i=1}^{n} [(x_i - \mu)^2 P(x_i)]$$

(5.2)

equivalent formula

$$\sigma^2 = \sum_{i=1}^{n} [x_i^2 P(x_i)] - \mu^2$$

(5.3b)
5: Probability Distributions (Discrete Variables)

5.2 Mean and Variance of a Discrete Random Variable

\[
\sigma^2 = \sum_{i=1}^{n} [(x_i - \mu)^2 P(x_i)] = (x_1 - \mu)^2 P(x_1) + (x_2 - \mu)^2 P(x_2) + \ldots + (x_n - \mu)^2 P(x_n)
\]

For the # of H when we flip a coin twice discrete distribution:

\[
\sigma^2 = (x_1 - \mu)^2 \cdot P(x_1) + (x_2 - \mu)^2 \cdot P(x_2) + (x_3 - \mu)^2 \cdot P(x_3)
\]

\[
\sigma^2 = (0-1)^2 \cdot P(0) + (1-1)^2 \cdot P(1) + (2-1)^2 \cdot P(2)
\]

\[
\sigma^2 = 1/4 + 0 + 1/4
\]

\[
\sigma^2 = 1/2 \quad \rightarrow \quad \sigma = \sqrt{\sigma^2} = 1/\sqrt{2}
\]
5: Probability Distributions (Discrete Variables)

5.3 The Binomial Probability Distribution

An experiment with only two outcomes is called a Binomial exp. Call one outcome *Success* and the other *Failure*. Each performance of expt. is called a trial and are independent.

\[ P(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \]

\( x = 0, \ldots, n \)

- \( n \): number of trials or times we repeat the experiment.
- \( x \): the number of successes out of \( n \) trials.
- \( p \): the probability of success on an individual trial.

\[ \binom{n}{x} = \frac{n!}{x!(n-x)!} \]

Bi means two like bicycle
5: Probability Distributions (Discrete Variables)

5.3 The Binomial Probability Distribution

Flip coin ten times.

\[
P(x) = \frac{n!}{x!(n-x)!} p^x (1 - p)^{n-x}
\]

\(n=10, \; x=7, \; p=.5\)

\[
P(7) = \frac{10!}{7!(10-7)!} \left(\frac{1}{2}\right)^7 \left(1 - \frac{1}{2}\right)^{10-7}
\]

\[
P(7) = \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7!3!} \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^{10-7}
\]

\[
P(7) = \frac{10 \cdot 3 \cdot 2 \cdot 4}{3 \cdot 2} \left(\frac{1}{2}\right)^{10} \rightarrow P(7) = \frac{120}{1024}
\]
## 5: Probability Distributions (Discrete Variables)

### 5.3 The Binomial Probability Distribution

The binomial probability distribution is given by the formula:

$$ P(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} $$

Where:
- $n$ is the number of trials,
- $p$ is the probability of success,
- $x$ is the number of successes.

For $n=10$ and $p=1/2$,

$$ P(x) = \frac{10!}{x!(10-x)!} \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{10-x} $$

### Table 2: Binomial Probabilities

<table>
<thead>
<tr>
<th>$n$</th>
<th>$x$</th>
<th>$P(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.904</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0.991</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0.0004</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0.0001</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0.00001</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0.0000001</td>
</tr>
</tbody>
</table>

The table continues with values for different $x$ values up to $10$. Each row represents a different $n$ value, and each column represents a different $x$ value.

Figure from Johnson & Kuby, 2012.
5: Probability Distributions (Discrete Variables)
5.3 The Binomial Probability Distribution

Example: \( n = 10, \; p = 1/2 \)
What is the probability of getting 4, 5, or 6 heads?

\[
P(4 \leq x \leq 6) = P(4) + P(5) + P(6)
\]

\[
P(4 \leq x \leq 6) = \frac{210}{1024} + \frac{252}{1024} + \frac{210}{1024}
\]

\[
P(4 \leq x \leq 6) = \frac{672}{1024} \approx 0.6123
\]

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(x) )</td>
<td>( \frac{1}{1024} )</td>
<td>( \frac{10}{1024} )</td>
<td>( \frac{45}{1024} )</td>
<td>( \frac{120}{1024} )</td>
<td>( \frac{210}{1024} )</td>
<td>( \frac{252}{1024} )</td>
<td>( \frac{210}{1024} )</td>
<td>( \frac{120}{1024} )</td>
<td>( \frac{45}{1024} )</td>
<td>( \frac{10}{1024} )</td>
<td>( \frac{1}{1024} )</td>
</tr>
</tbody>
</table>
5: Probability Distributions (Discrete Variables)

5.3 Mean and Standard Deviation of the Binomial Distribution

The formula for the mean $\mu$ and variance $\sigma^2$ of Binomial is

$$
\mu = \sum_{x=0}^{n} x \frac{n!}{x! (n-x)!} p^x (1 - p)^{n-x}
$$

$$
= np
$$

(5.7)

$$
\sigma^2 = \sum_{x=0}^{n} (x - \mu)^2 \frac{n!}{x! (n-x)!} p^x (1 - p)^{n-x}
$$

$$
= np(1 - p) \quad \longrightarrow \quad \sigma = \sqrt{np(1 - p)}
$$

(5.8)
5: Probability Distributions (Discrete Variables)
5.3 Mean and Standard Deviation of the Binomial Distribution

Example:
Before, using $\mu = \sum_{x=0}^{n} [xP(x)]$, we found $\mu = 1$.

Now using $\mu = np$, we get $\mu = (2) \cdot (1/2) = 1$.

Before, using $\sigma^2 = \sum_{x=0}^{n} [(x - \mu)^2 P(x)]$, we found $\sigma^2 = 1/2$.

Now using $\sigma^2 = np(1 - p)$, we get $\sigma^2 = (2) \cdot (1/2) \cdot (1/2) = 1/2$.