Class 23

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Department of Mathematics, Statistics, and Computer Science
Agenda:
Course Discussion
Recap Chapter 9.1
Lecture Chapter 9.2 and 9.3
Course evaluations begin April 23

Spring 2012 course evaluations will be available for students to complete online from Monday, April 23, through Sunday, May 6. The Marquette Online Course Evaluations System will be used to administer the evaluations.

Students will receive an e-mail to their eMarq e-mail account Monday, April 22, with login information and instructions about how to complete the evaluations. Students can also access the online system directly.
http://www.marquette.edu/evaluate/

Results will be made available to faculty after all final grades for all classes have been submitted to the Office of the Registrar, anticipated to be Wednesday, May 23.
Recap Chapter 9.1
9: Inferences Involving One Population
9.1 Inference about the Mean $\mu$ ($\sigma$ Unknown)

In Chapter 8, we performed hypothesis tests on the mean by

1) assuming that $\bar{x}$ was normally distributed ($n$ “large”),

2) assuming the hypothesized mean $\mu_0$ were true,

3) assuming that $\sigma$ was known, so that we could form

$$z^* = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

which with 1) – 3) has standard normal dist.
9: Inferences Involving One Population
9.1 Inference about the Mean $\mu$ ($\sigma$ Unknown)

However, in real life, we never know $\sigma$ for

$$z^* = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

so we would like to estimate $\sigma$ by $s$, then use

$$t^* = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}.$$ 

But $t^*$ does not have a standard normal distribution.

It has what is called a Student $t$-distribution.
9: Inferences Involving One Population
9.1 Inference about the Mean $\mu$ ($\sigma$ Unknown)
Using the $t$-Distribution Table

Finding critical value from a Student $t$-distribution, $df = n - 1$

$t(df, \alpha)$, $t$ value with $\alpha$ area larger than it

with $df$ degrees of freedom

Table 6
Appendix B
Page 719.

Figure from Johnson & Kuby, 2012.
9: Inferences Involving One Population

9.1 Inference about the Mean \( \mu \) (\( \sigma \) Unknown)

Example: Find the value of \( t(10,0.05) \), \( df=10, \alpha=0.05 \).

Area in One Tail

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<th>0.25</th>
<th>0.10</th>
<th>0.05</th>
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Area in Two Tails

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Table 6
Appendix B
Page 719.

Go to 0.05 One Tail column and down to 10 df row.

Figures from Johnson & Kuby, 2012.

Rowe, D.B.
9: Inferences Involving One Population
9.1 Inference about the Mean $\mu$ ($\sigma$ Unknown)

Recap 9.1:
Essentially have new critical value, $t(df, \alpha)$ to look up
in a table when $\sigma$ is unknown. Used same as before.

\[
\sigma \text{ assumed known} \quad \text{vs.} \quad \sigma \text{ assumed unknown}
\]

\[
\bar{x} \pm z(\alpha / 2) \frac{\sigma}{\sqrt{n}} \quad \rightarrow \quad \bar{x} \pm t(df, \alpha / 2) \frac{s}{\sqrt{n}}
\]

\[
z^* = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \quad \rightarrow \quad t^* = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}
\]
Chapter 9: Inferences Involving One Population

Questions?

Homework: Chapter 9 # 7, 21, 23, 35, 37, 39, 47, 55, 67, 73, 75, 93, 95, 97, 103, 117, 119, 121, 129, 131, 135
Chapter 9: Inferences Involving One population (continued)

Daniel B. Rowe, Ph.D.

Department of Mathematics, Statistics, and Computer Science
We talked about a Binomial experiment with two outcomes. Each performance of the experiment is called a trial. Each trial is independent.

\[
P(x) = \frac{n!}{x!(n-x)!} p^x (1 - p)^{n-x}
\]

\begin{align*}
& n = 1, 2, 3, \ldots \\
& 0 \leq p \leq 1 \\
& x = 0, 1, \ldots, n
\end{align*}

\(n\) = number of trials or times we repeat the experiment. \\
\(x\) = the number of successes out of \(n\) trials. \\
\(p\) = the probability of success on an individual trial.
When we perform a binomial experiment we can estimate the probability of heads as

\[
p' = \frac{x}{n}
\]

(9.3)

where \( x \) is the number of successes in \( n \) trials.

This is a point estimate. Recall the rule for a CI is

point estimate \( \pm \) some amount
### 9: Inferences Involving One Population

#### 9.2 Inference about the Binomial Probability of Success

**Background**

In Statistics, if we have a random variable $x$ with

\[
\text{mean}(x) = \mu \quad \text{and} \quad \text{variance}(x) = \sigma^2
\]

then the mean and variance of $cx$ where $c$ is a constant is

\[
\text{mean}(cx) = c\mu \quad \text{and} \quad \text{variance}(cx) = c^2\sigma^2.
\]

If $x$ has a binomial distribution then

\[
\text{mean}(cx) = cnp \quad \text{and} \quad \text{variance}(cx) = c^2np(1 - p).
\]

This is a rule.
With \( p' = \frac{x}{n} \), the constant is \( c = \frac{1}{n} \), and

\[
\text{mean} \left( \frac{x}{n} \right) = \left( \frac{1}{n} \right) \text{mean}(x) = \left( \frac{1}{n} \right) np = p
\]

so the variance of \( p' = \frac{x}{n} \) is

\[
\text{variance} \left( \frac{x}{n} \right) = \frac{\sigma^2}{n^2} = \frac{p(1-p)}{n}
\]

standard error of \( p' = \frac{x}{n} \) is

\[
\sigma_{p'} = \sqrt{\frac{\sigma^2}{n^2}} = \sqrt{\frac{p(1-p)}{n}}
\]
That is where 1. and 2. in the green box below come from

If a random sample of size $n$ is selected from a large population with $p = P(\text{success})$, then the sampling distribution of $p'$ has:

1. A mean $\mu_p$, equal to $p$

2. A standard error $\sigma_{p'}$, equal to $\sqrt{\frac{p(1-p)}{n}}$

3. An approximately normal distribution if $n$ is sufficiently “large.”
If we flip the coin a large number of times

\[ P(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \]

\[ x = 0, \ldots, n \]

\( x \) = \# of heads when we flip a coin \( n \) times

\( n = 14 \)
\( p = 1/2 \)

It gets tedious to find the \( n=14 \) probabilities!

Figure from Johnson & Kuby, 2012.
6: Normal Probability Distributions
6.5 Normal Approximation of the Binomial Distribution

It gets tedious to find the $n=14$ probabilities!

So what we can do is use a histogram representation,

Figures from Johnson & Kuby, 2012.
6: Normal Probability Distributions
6.5 Normal Approximation of the Binomial Distribution

So what we can do is use a histogram representation, \( n=14 \), \( p=1/2 \)

Then approximate binomial probabilities with normal areas.

Figures from Johnson & Kuby, 2012.
Approximate binomial probabilities with normal areas. Use a normal with $\mu = np$, $\sigma^2 = np(1 - p)$

$\mu = (14)(.5) = 7$

$\sigma^2 = (14)(.5)(1 - .5) = 3.5$

Figures from Johnson & Kuby, 2012.
We then approximate binomial probabilities with normal areas.

\[ P(x = 4) \text{ from the binomial formula} \]

is approximately \( P(3.5 < x < 4.5) \)

from the normal with \( \mu = 7, \sigma^2 = 3.5 \)

\[ \text{the ±}.5 \text{ is called a "continuity correction"} \]

Figures from Johnson & Kuby, 2012.
9: Inferences Involving One Population
9.2 Inference about the Binomial Probability of Success

In practice, using these guidelines will ensure normality of $x$:  

1. The sample size $n$ is greater than 20.  
2. The product $np$ and $n(1-p)$ are both greater than 5.  
3. The sample consists of less than 10% of the population.  

1. $n \geq 20$, 2. $np \geq 5$ and $n(1-p) \geq 5$, 3. $\frac{n}{N} < .10$.  

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9: Inferences Involving One Population
9.2 Inference about the Binomial Probability of Success

But we’re not using $x$, we’re scaling it and using $p' = \frac{x}{n}$.

It turns out that $p' = \frac{x}{n}$ also has an approx. normal distribution.

$$\mu_x = np \quad \sigma_x^2 = np(1-p)$$

$$\mu_{p'} = p \quad \sigma_{p'}^2 = \frac{p(1-p)}{n}$$
9: Inferences Involving One Population

9.2 Inference about the Binomial Probability of Success

Now we can determine probabilities with normal areas.

\[ P(3.5 \leq x \leq 4.5) = 0.0594 \]

\[ P(3.5/14 \leq p' \leq 4.5/14) = 0.0594 \]

\[ n=14, \quad p=1/2 \]

\[ p' = \frac{x}{n} \]

\[ \mu_{p'} = 0.5, \quad \sigma^2_{p'} = 0.018 \]

Need to convert to \( z \)'s.

Figure left from and right modified Johnson & Kuby, 2012.
9: Inferences Involving One Population
9.2 Inference about the Binomial Probability of Success

Now we can determine probabilities with normal areas.

For $x$

$P(3.5 < x < 4.5)$

$P(3.5 - 7 < x - np < 4.5 - 7)$

$P\left(\frac{3.5 - 7}{\sqrt{3.5}} < \frac{x - np}{\sqrt{np(1 - p)}} < \frac{4.5 - 7}{\sqrt{3.5}}\right)$

Now we can look up areas.

For $p'$

$P(.25 < p' < .32)$

$P(.25 - .5 < p' - p < .32 - .5)$

$P\left(\frac{.25 - .5}{\sqrt{.5(1-.5)}} < \frac{p' - p}{\sqrt{p(1-p)}} < \frac{.32 - .5}{\sqrt{.5(1-.5)}}\right)$
9: Inferences Involving One Population
9.2 Inference about the Binomial Probability of Success

For a confidence interval, we would use

Confidence Interval for a Proportion

\[ p' - z(\alpha / 2)\sqrt{\frac{p'q'}{n}} \quad \text{to} \quad p' + z(\alpha / 2)\sqrt{\frac{p'q'}{n}} \]  

(9.6)

where \( p' = \frac{x}{n} \) and \( q' = (1 - p') \).

Since we didn’t know the true value for \( p \), we estimate it by \( p' \).

This is of the form point estimate \( \pm \) some amount.
9: Inferences Involving One Population
9.2 Inference about the Binomial Probability of Success

Example:
Dana randomly selected \( n = 200 \) cars and found \( x = 17 \) convertibles. Find the 90\% CI for the proportion of cars that are convertibles.

\[
p' = \frac{x}{n} = \frac{17}{200}
\]
\[
\alpha = 0.1
\]
\[
z(\alpha / 2) = z(0.1 / 2) = 1.65
\]
\[
p' \pm z(\alpha / 2) \sqrt{\frac{p'q'}{n}}
\]
\[
\frac{17}{200} \pm 1.65 \sqrt{\frac{(17/200)(1-17/200)}{200}}
\]
\[
0.052 \text{ to } 0.118
\]
9: Inferences Involving One Population

9.2 Inference about the Binomial Probability of Success

Determining the Sample Size

Using the error part of the CI, we determine the sample size \( n \).

Maximum Error of Estimate for a Proportion

\[
E = z\left(\frac{\alpha}{2}\right) \sqrt{\frac{p'(1-p')}{n}}
\]

(9.7)

Sample Size for 1- \( \alpha \) Confidence Interval of \( p \)

\[
n = \frac{\left[z\left(\frac{\alpha}{2}\right)\right]^2 p* (1 - p*)}{E^2}
\]

(9.8)

From prior data, experience, gut feelings, séance. Or use 1/2.

where \( p^* \) and \( q^* \) are provisional values used for planning.
9: Inferences Involving One Population
9.2 Inference about the Binomial Probability of Success
Determining the Sample Size

Example:
A supplier claims bolts are approx. 5% defective. Determine the sample size \( n \) if we want our estimate within ±0.02 with 90% confidence.

\[
E = 0.02 \quad p^* = 0.05
\]

\[
z(0.1 / 2) = 1.65
\]

\[
1 - \alpha = 0.90
\]

\[
n = \frac{[z(\alpha / 2)]^2 p^*(1 - p^*)}{E^2}
\]

\[
n = \frac{[1.65]^2 (0.05)(1 - 0.05)}{(0.02)^2} = \frac{0.12931875}{0.0004} = 323.4 \rightarrow n = 324
\]
We can perform hypothesis tests on the proportion

\[ H_0: p \geq p_0 \text{ vs. } H_a: p < p_0 \]
\[ H_0: p \leq p_0 \text{ vs. } H_a: p > p_0 \]
\[ H_0: p = p_0 \text{ vs. } H_a: p \neq p_0 \]

**Test Statistic for a Proportion \( p \)**

\[
z^* = \frac{p' - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \quad \text{with} \quad p' = \frac{x}{n}
\]

(9.9)
9: Inferences Involving One Population
9.3 Inference about the Binomial Probability of Success

Example:
Reported that 61% get > 7 hrs of sleep per night on weekend. A sample \( n=350 \) found that \( x=235 \) had > 7 hours sleep.

With \( \alpha=.05 \), does evidence show > 61% sleep > 7 hrs on weekend?

\[
H_0: p = .61 \ (\leq) \ vs. \ H_a: p > .61
\]

\[
p' = \frac{x}{n} = \frac{235}{350} = 0.671
\]

\[
z^* = \frac{p' - p_0}{\sqrt{p_0(1-p_0)/n}} = \frac{0.671 - 0.61}{\sqrt{0.61(1-0.61)/350}} = 2.34
\]

\( \alpha=0.05 \)
9: Inferences Involving One Population
9.3 Inference about the Binomial Probability of Success

Example:

\[ H_0: p = 0.61 \ (\leq) \ vs. \ H_a: p > 0.61 \]

Find the critical value(s).
\[ \alpha = 0.05 \]
\[ P(z > z(0.05)) = 1.65 \]

Since \( 2.34 > 1.65 \), Reject \( H_0 \).

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9: Inferences Involving One Population

9.3 Inference about the Variance and Standard Deviation

We can perform hypothesis tests on the variance.

\[ H_0: \sigma^2 \geq \sigma_0^2 \quad \text{vs.} \quad H_a: \sigma^2 < \sigma_0^2 \]

\[ H_0: \sigma^2 \leq \sigma_0^2 \quad \text{vs.} \quad H_a: \sigma^2 > \sigma_0^2 \]

\[ H_0: \sigma^2 = \sigma_0^2 \quad \text{vs.} \quad H_a: \sigma^2 \neq \sigma_0^2 \]

The assumptions for inferences about the variance \( \sigma^2 \) or standard deviation \( \sigma \):

The sampled population is normally distributed.
9: Inferences Involving One Population
9.1 Inference about the Mean \( \mu \) (\( \sigma \) Unknown)

What is the Student \( t \)-distribution and how do we get it?

Background Information

If the data comes from normally distributed population, then

\[
x \sim N(\mu, \sigma^2)
\]

mean variance

\[
\bar{x} \sim N(\mu, \sigma^2 / n)
\]

mean variance

generate \( 5 \times 10^6 \) random values \( \mu = 100 \)

\( \sigma = 57.7 \)

\( n = 5 \)

\( n = 5 \)

\( 5 \times 10^6 \)

\( 1 \times 10^6 \)

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9: Inferences Involving One Population
9.1 Inference about the Mean $\mu$ ($\sigma$ Unknown)

It turns out that with the variance $\sigma^2$ known, the distribution of
\[
\frac{(n-1)s^2}{\sigma^2}
\]
has a chi-square distribution with $n-1$ degrees of freedom.

(\chi^2 distribution on Pages 453-454)
9: Inferences Involving One Population
9.3 Inference about the Variance and Standard Deviation

Properties of the chi-square distribution

1. $\chi^2$ is nonnegative
2. $\chi^2$ is not symmetric, skewed to right
   mode $<$ median $<$ mean
3. $\chi^2$ is distributed to form a family each determined by
   $df = \nu = n - 1$. 

$$\text{median} \approx \nu - \frac{2}{3} + \frac{4}{27\nu} - \frac{8}{729\nu^2}$$

$$\text{mode} = \nu - 2, \quad \nu > 2$$

$$f(\chi^2 | \nu) = \frac{(\chi^2)^{\nu/2-1} e^{-\chi^2/2}}{\Gamma(\nu/2) 2^{\nu/2}}$$

$\mu = \nu$

$\sigma^2 = 2\nu$

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9: Inferences Involving One Population
9.3 Inference about the Variance and Standard Deviation

Test Statistic for Variance (and Standard Deviation)

\[
\chi^2* = \frac{(n - 1)s^2}{\sigma_0^2}, \quad \text{with } df=n-1. \tag{9.10}
\]

Will also need critical values.

\[
P(\chi^2 > \chi^2(df, \alpha)) = \alpha
\]

Table 8
Appendix B
Page 721

Figure from Johnson & Kuby, 2012.
9: Inferences Involving One Pop.

Example: Find $\chi^2(20,0.05)$.

Table 8, Appendix B, Page 721.

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Figures from Johnson & Kuby, 2012.

Rowe, D.B.
9: Inferences Involving One Population

Example:
Soft-drink bottling company wants to control variability by not allowing the variance to exceed 0.0004. Sample taken, \( n=28 \), \( s^2=0.0007 \) and \( \alpha=0.05 \).

Step 1

\[ H_0: \sigma^2 \leq 0.0004 \text{ vs. } H_a: \sigma^2 > 0.0004 \]

Step 2

\[ \chi^2* = \frac{(n-1)s^2}{\sigma_0^2} \]

\[ df=n-1 \]

Step 3

\[ \chi^2* = \frac{(28-1)(0.0007)}{0.0004} = 47.25 \]

Step 4

0.005 < \text{p-value} < 0.01 and \( \chi^2(df, \alpha) = 40.1 \)

Step 5

Reject the null hypothesis.
Chapter 9: Inferences Involving One Population

Questions?

Homework: Chapter 9 # 5, 7, 9, 21, 23, 27, 28, 35, 37, 43, 49, 55, 71, 85, 89, 91, 93, 95, 97, 99, 105, 109, 119, 121, 129, 131, 135, 139, 145