Agenda:

Recap Chapter 4.4 - 4.6

Lecture Chapter 5.1 - 5.4

TA Go Over Exam 2.
Recap Chapter 4.4 - 4.6
4: Probability
4.4 Rules of Probability

Union Example \((A \text{ or } B)\): Rolling a single die.
\(A=\text{event } \#1,2,3. \ B=\text{event odd number.}\)

\((A \text{ or } B)\)

\[
P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)
\]

\(S\)

\(1 \hspace{0.5cm} 2 \hspace{0.5cm} 3 \hspace{0.5cm} 4 \hspace{0.5cm} 5 \hspace{0.5cm} 6\)

\(A\)

\[
\begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 & 6 \\
\hline
0 & 0 & 1 & 0 & 0 & 0 \\
\end{array}
\]

double count 1 and 3

\(B\)

\[
\begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 & 6 \\
\hline
1 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

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4: Probability
4.4 Rules of Probability

**Intersection Example** *(A and B)*: Rolling a single die. 
*A* = event #1, 2, 3. 
*B* = event odd number. 
*(A and B) = {1, 3}*

\[
P(A \text{ and } B) = P(A) \cdot P(B \mid A)
\]

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4: Probability
4.5 Mutually Exclusive Events

Mutually Exclusive Example: Rolling a single die. 
$A$=event #1,2. $B$=event #5,6.

$A$ and $B = \emptyset$

$P(A \text{ and } B) = 0$

$S$ no overlap
Independent events: … the occurrence or nonoccurrence of one gives no information about … occurrence for the other.

\[ P(A) = P(A \mid B) = P(A \mid \text{not } B) \]

Dependent events: … occurrence of one event does have an effect on the probability of occurrence of the other event.

\[ P(A) \neq P(A \mid B) \]

Special multiplication rule:
Let \( A \) and \( B \) … independent events … in a sample space \( S \).

\[ P(A \text{ and } B) = P(A) \cdot P(B) \]
4: Probability

Questions?

Homework: Chapter 4 # 57, 61, 63, 67, 87, 91, 95, 103, 105, 111
Lecture Chapter 5.1 - 5.4
Chapter 5: Probability Distributions (Discrete Variables)

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5: Probability Distributions (Discrete Variables)
5.2 Random Variables

**Random Variables:** A variable that assumes a unique numerical value for each of the outcomes in the sample space of a probability experiment.

Example:
Let $x$ = the number of heads when we flip a coin twice.

$x = \{0, 1, 2\}$ ← numerical values for

$\{TT, TH, HT, HH\}$ ← outcomes in sample space
5: Probability Distributions (Discrete Variables)

5.2 Random Variables

**Discrete Random Variables:** A quantitative random variable that can assume a countable number of values.

**Continuous Random Variable:** A quantitative random variable that can assume an uncountable number of values.

**Examples:**
- **Discrete:** Number of heads when we flip a coin ten times.
- **Continuous:** Distance from earth center to sun center.
Probability Distribution: A distribution of the probabilities associated with each of the values of a random variable. The probability distribution is a theoretical distribution; it is used to represent populations.

**TABLE 5.1**

<table>
<thead>
<tr>
<th>Probability Distribution: Tossing Two Coins</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$x = # H$</td>
<td>$P(x)$</td>
</tr>
<tr>
<td>0</td>
<td>0.25</td>
</tr>
<tr>
<td>1</td>
<td>0.50</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
</tr>
</tbody>
</table>

**TABLE 5.2**

<table>
<thead>
<tr>
<th>Probability Distribution: Rolling a Die</th>
<th>$x$=face value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>1</td>
</tr>
<tr>
<td>$P(x)$</td>
<td>1</td>
</tr>
</tbody>
</table>

Recall: 1. each prob between 0 & 1 and 2. sum of prob’s = 1

Figures from Johnson & Kuby, 2008.
5: Probability Distributions (Discrete Variables)
5.3 Probability Distributions of a Discrete Random Variable

**Probability Function:** A rule \( P(x) \) that assigns probabilities to the values of the random variable \( x \).

**Example:**
Let \( x = \) # of heads when we flip a coin twice.

\( x = \{0,1,2\} \)

\[
P(x) = \frac{2!}{x!(2-x)!} \left( \frac{1}{2} \right)^x \left( \frac{1}{2} \right)^{2-x}
\]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( P(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( \frac{1}{4} )</td>
</tr>
<tr>
<td>1</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{1}{4} )</td>
</tr>
</tbody>
</table>
5: Probability Distributions (Discrete Variables)

5.3 Probability Distributions of a Discrete Random Variable

Example:
Let $x = \# \text{ of H when flip a coin twice.}$

$$P(x) = \frac{2!}{x!(2-x)!} \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{2-x}$$
for $x = 0, 1, 2$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$P(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>1</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{1}{4}$</td>
</tr>
</tbody>
</table>

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1: Statistics
1.2 What is Statistics?

**Data:** The set of values collected from the variable from each of the elements that belong to the sample.

**Experiment:** A planned activity whose results yield a set of data.

**Sample:** Subset of the population.

**Parameter:** A numerical value summarizing all the data of an entire population.

**Statistic:** A numerical value summarizing the sample data.
5: Probability Distributions (Discrete Variables)
5.4 Mean and Variance of a Discrete Random Variable

If we calculate a numerical summary from the sample of data, it is called a **statistic**. i.e. $\bar{x}$ and $s^2$.

If we calculate a numerical summary from the population of data, it is called a **parameter**. i.e. $\mu$ and $\sigma^2$.

$\mu$ is the Greek letter lower case mu.

$\sigma$ is the Greek letter lower case sigma.
5: Probability Distributions (Discrete Variables)
5.4 Mean and Variance of a Discrete Random Variable

Mean of a discrete random variable (expected value):
The mean, $\mu$, of a discrete random variable $x$ is found by multiplying each possible value of $x$ by its own probability $P(x)$ and then adding all of the products together:

$$\mu = \sum_{i=1}^{n} [x_i P(x_i)]$$

(5.1)
5: Probability Distributions (Discrete Variables)
5.4 Mean and Variance of a Discrete Random Variable

\[ \mu = \sum_{i=1}^{n} [x_i P(x_i)] = x_1 P(x_1) + x_2 P(x_2) + \ldots + x_n P(x_n) \]

For the discrete distribution we just discussed it is:

\[ \mu = (0) \cdot P(0) + (1) \cdot P(1) + (2) \cdot P(2) \]

\[ \mu = (0) \cdot (1/4) + (1) \cdot (1/2) + (2) \cdot (1/4) \]

\[ \mu = 0 + 1/2 + 1/2 \]

\[ \mu = 1 \]
5: Probability Distributions (Discrete Variables)
5.4 Mean and Variance of a Discrete Random Variable

Variance of a discrete random variable: The variance, $\sigma^2$, of a discrete random variable $x$ is found by multiplying each possible value of the squared deviation, $(x - \mu)^2$, by its own probability $P(x)$ and then adding all of the products together:

$$\sigma^2 = \sum_{i=1}^{n} [(x_i - \mu)^2 P(x_i)]$$

(5.2)

variance of $x$: sigma squared

= sum of (squared deviation times probability)

equivalent formula

$$\sigma^2 = \sum_{i=1}^{n} [x_i^2 P(x_i)] - \mu^2$$

(5.3b)
5: Probability Distributions (Discrete Variables)

5.4 Mean and Variance of a Discrete Random Variable

For the discrete distribution we just discussed it is:

$$\sigma^2 = \sum_{i=1}^{n} [(x_i - \mu)^2 P(x_i)] = (x_1 - \mu)^2 P(x_1) + (x_2 - \mu)^2 P(x_2) + \ldots + (x_n - \mu)^2 P(x_n)$$

For the discrete distribution we just discussed it is:

$$\sigma^2 = (0-1)^2 \cdot P(0) + (1-1)^2 \cdot P(1) + (2-1)^2 \cdot P(2)$$

$$\sigma^2 = (-1)^2 \cdot (1/4) + (0)^2 \cdot (1/2) + (1)^2 \cdot (1/4)$$

$$\sigma^2 = 1/4 + 0 + 1/4$$

$$\sigma^2 = 1/2$$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$P(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$1/4$</td>
</tr>
<tr>
<td>$x_1$</td>
<td>1</td>
</tr>
<tr>
<td>$x_2$</td>
<td>2</td>
</tr>
<tr>
<td>$x_3$</td>
<td>$1/4$</td>
</tr>
</tbody>
</table>

$\mu = 1$

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5: Probability Distributions (Discrete Variables)

5.4 Mean and Variance of a Discrete Random Variable

**Standard deviation of a discrete variable:** The positive square root of the variance.

\[
\sigma = \sqrt{\sigma^2}
\]

\[
\sigma = \sqrt{\sum_{i=1}^{n} [(x_i - \mu)^2 P(x_i)]}
\]

\[
\sigma^2 = 1/2
\]

\[
\sigma = 1/\sqrt{2}
\]
5: Probability Distributions (Discrete Variables)

Questions?

Homework: Chapter 5 # 17, 19, 31, 35, 37, 45, 57, 91, 93
Go over Exam 2, TA.