Probabilistic Forecasting
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Introduction
Probabilistic forecasting is increasingly becoming a useful tool for forecasting when taking account of uncertainty is of great concern. Much work has been done on probabilistic forecasting on various fields [2]. Therefore it is of paramount importance to have the proper tools to distinguish between good and bad probabilistic forecasts. Furthermore, much confusion can arise when working with probabilistic forecasts communicating the results. Hence the need for a clear unambiguous definition of what a probabilistic forecast is. In addition, it is helpful to compare different scoring metrics for probabilistic forecasts.

Properties of Probabilistic Forecasts
One way to assess calibration is with the probability integral transform (PIT) of the CDF. The restriction to density forecasts can be impractical, and we now discuss proper scoring rules that are indicated in forecasts in the prevalent westerly regime. Much work has been done on probabilistic forecasting on various fields [2].

CDF Properties
One way to assess calibration is with the probability integral transform (PIT) of the CDF.

Scoring Metrics:
Continuous Ranked Probability Score
The continuous ranked probability score (CRPS) is defined as:

$$\text{CRPS}(F, y) = \int_{-\infty}^{\infty} (F(x) - I(y \leq x))^2 \, dx$$

where Y and Y' are independent random variables with CDF F and finite first moment (Gneiting & Raftery 2007). Matheson & Winkler (1976).

Dawid–Sebastiani Score
The CRPS has many attractive properties, but it can be hard to compute for complex forecast distributions. A viable alternative that depends on the probabilistic forecast, F, through only its first two central moments, µ and σ, is given by the proper Dawid-Sebastiani score (DSS) (Dawid & Sebastiani 1999).

$$\text{DSS}(F, y) = \frac{(y - \mu)^2}{\sigma^2} + 2 \log \Phi(y)$$

Pinball Score
For a quantile forecast qα at the α target quantile and having y as the realized value the Pinball score L(qα, y) is defined as:

$$L(q_\alpha, y) = \begin{cases} 1 - \Phi(y - q_\alpha) & y < q_\alpha \\ \Phi(y - q_\alpha) & y \geq q_\alpha \end{cases}$$

Future Work
• Try to find weaknesses in the scoring metrics that could be exploited.
• Create a probabilistic forecast engine to test different scoring metrics against each other.
• Create a machine learning model that tries to improve the score of a probabilistic forecast engine.
• Further research into probabilistic forecast ensembles might be helpful [3].

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References

Figure 1: Two-hour-ahead regime-switching space-time forecasts of hourly average wind speed at the Stateline wind energy center for the 7-day period beginning July 5, 2003. The predictive mean is shown in green, the 90% central prediction interval in red, and the realized wind speed in orange. The blue marks at the top indicate forecasts in the prevalent westerly regime.

Figure 2: (a) Probability integral transform (PIT) histogram, (b) sample autocorrelation function (ACF) for the PIT values, and (c) calibration curve for exceedance of 10 m/s with bootstrap confidence intervals for two-hour-ahead regime-switching space-time forecasts of hourly average wind speed at the Stateline wind energy center.

Figure 3: This model produces a distribution of outcomes for the national popular vote. The curves will get narrower as the election gets closer and the forecasts become more confident. [1]

Figure 4: These win probabilities come from simulating the election 20,000 times, which produces a distribution of possible outcomes for each state. The closer the dot is to the center line, the tighter the race. And the wider the bar, the less certain the model is about the outcome. [3]