Graph Theory: Software Design for Modeling Competition and (i,j)-Step Competition Graphs with Predator-Prey Applications in an Ecosystem

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BACKGROUND

Graphs are a useful applications for studying predator-prey relations between species in an ecosystem.

Food webs are one example of a graph which maps the flow of energy in an ecosystem by illustrating the feeding relationships between species. They are used to display the dependency from one organism to the other in an environment.

To further model the predator-prey relationship, Cohen introduced the competition graph of a food web in 1968. The competition graph demonstrates the direct competition between various species.

Factor and Merz extended this theory, through the introduction of the (i,j)-step competition graph in 2010. The (i,j)-step competition graph displays both direct and indirect competition between species, giving a broader view of specie relations.

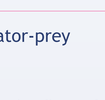
Each of these graphs are tools to understand how the ecosystem may respond to change or what controlled changes can be made in order to obtain desired properties in an ecosystem.

DEFINITIONS

Definition: *A digraph is a set of vertices V(D) and a set of arcs A(D). An arc (u,v) is represented by an arrow from vertex u to vertex v.*

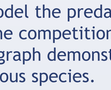
Definition: *A biological digraph representation of a food web shows the energy flow, from prey to predator. The direction of the arrows point upward. This is the transposition of the mathematical model.*

Definition: *A mathematical digraph representation of a food web shows the arrows pointed downwards from predator to prey. This is the transposition of the biological model.*



Definition: *Given a digraph D, the competition graph of D, denoted C(D), is the graph with the same vertex set as D and an edge between vertices u and v if and only if $O^+(x) \cap O^+(y) \neq 0$.*

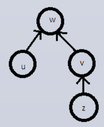
Digraph D:



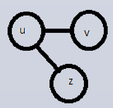
Competition Graph C(D):

Definition: *Given a digraph D, the (i,j)-step competition graph of D, denoted $C_{i,j}(D)$, is the graph with the same vertex set as D and an edge between vertices u and v if and only if there exist some $z \in V(G)$ for either $dD/\{u\}(v,z) = i$ and $dD\{v\}(u,z) <= j$ or $dD/\{v\}(u,z) = j$ and $dD\{u\}(v,z) <= i$*

Digraph D:

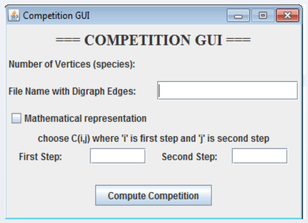


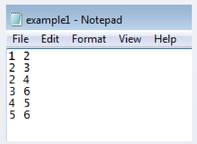
Competition Graph $C_{1,2}(D)$:



MATERIALS AND METHODS

The Graphical User Interface (GUI) uses a text file that lists the edges in a digraph to create the desired competition graphs:



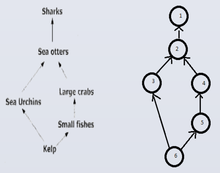


**file "example1.txt" represents the digraph in Example A below

The program uses the edges to build an adjacency matrix, also known as the Digraph Matrix, which is used to generate the possible competition and step competition pathways.

Below are three examples of food webs representing the acyclic digraphs used as inputs for the program to create competition graphs along with step-competition graphs.

Example A.



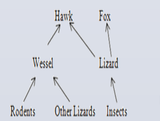
Biological Digraph Matrix Representation

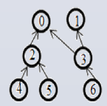
In/Out	1	2	3	4	5	6
1	0	1	0	0	0	0
2	0	0	1	1	0	0
3	0	0	0	0	0	1
4	0	0	0	0	1	0
5	0	0	0	0	0	0
6	0	0	0	0	0	0

Mathematical Digraph Matrix Representation

In/Out	1	2	3	4	5	6
1	0	0	0	0	0	0
2	1	0	0	0	0	0
3	0	1	0	0	0	0
4	0	1	0	0	0	0
5	0	0	0	1	0	0
6	0	0	1	0	0	0

Example B.





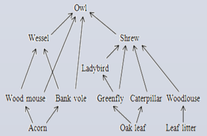
Biological Digraph Matrix Representation

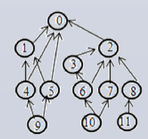
In/Out	0	1	2	3	4	5	6
0	0	0	1	1	0	0	0
1	0	0	0	1	0	0	0
2	0	0	0	0	1	1	0
3	0	0	0	0	0	0	1
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0

Mathematical Digraph Matrix Representation

In/Out	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0
3	1	1	0	0	0	0	0
4	0	0	1	0	0	0	0
5	0	0	1	0	0	0	0
6	0	0	0	1	0	0	0

Example C.





Biological Digraph Matrix Representation

In/out	0	1	2	3	4	5	6	7	8	9	10	11
0	0	1	1	0	1	1	0	0	0	0	0	0
1	0	0	0	0	1	1	0	0	0	0	0	0
2	0	0	0	1	0	0	1	1	1	0	0	0
3	0	0	0	0	0	0	1	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0	1	0	0
6	0	0	0	0	0	0	0	0	0	0	1	0
7	0	0	0	0	0	0	0	0	0	0	1	0
8	0	0	0	0	0	0	0	0	0	0	0	1
9	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0

Mathematical Digraph Matrix Representation

In/out	0	1	2	3	4	5	6	7	8	9	10	11
0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0
3	0	0	1	0	0	0	0	0	0	0	0	0
4	1	1	0	0	0	0	0	0	0	0	0	0
5	1	1	0	0	0	0	0	0	0	0	0	0
6	0	0	1	1	0	0	0	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0	0	0	0
8	0	0	1	0	0	0	0	0	0	0	0	0
9	0	0	0	0	1	1	0	0	0	0	0	0
10	0	0	0	0	0	0	1	1	0	0	0	0
11	0	0	0	0	0	0	0	0	1	0	0	0

RESULTS

Below are the outputs produced by the Competition GUI for the three digraph examples. The competitions chosen to display below include:

- Competition Graph
- C(1,2) – 1,2-Step Competition
- C(1,3) – 1,3-Step Competition
- C(2,2) – 2,2-Step Competition

Example A.

Example A -- Competition Graph

In/Out	1	2	3	4	5	6
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	1	0	1
4	0	0	1	0	1	0
5	0	0	0	0	0	1
6	0	0	0	0	0	0

Example A – C(1,2)

In/Out	1	2	3	4	5	6
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	1	1	0
4	0	0	1	0	0	1
5	0	0	1	0	0	0
6	0	0	0	1	0	0

Example A – C(1,3)

In/Out	1	2	3	4	5	6
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	1	1	1
4	0	0	1	0	0	1
5	0	0	1	0	0	0
6	0	0	1	1	0	0

Example A – C(2,2)

In/Out	1	2	3	4	5	6
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	1	1	0
4	0	0	1	0	0	1
5	0	0	1	0	0	0
6	0	0	0	1	1	0

Example B.

Example B – Competition Graph

In/Out	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	0	1	0	0	0
3	0	0	1	0	0	0	0
4	0	0	0	0	0	1	0
5	0	0	0	0	1	0	0
6	0	0	0	0	0	0	0

Example B – C(1,2)

In/Out	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	0	1	0	0	1
3	0	0	1	0	1	1	0
4	0	0	0	1	0	0	0
5	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0

Example B – C(1,3)

Step Size is too large

Example B – C(2,2)

In/Out	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	0	1	0	0	1
3	0	0	1	0	1	1	0
4	0	0	0	1	0	0	1
5	0	0	0	1	0	0	1
6	0	0	1	0	1	1	0

Example C.

Example C – Competition Graph

in/out	0	1	2	3	4	5	6	7	8	9	10	11
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	1	1	0	0	0	0	0	0
2	0	1	0	0	1	1	0	0	0	0	0	0
3	0	0	0	0	0	0	1	1	1	0	0	0
4	0	1	1	0	0	1	0	0	0	0	0	0
5	0	1	1	0	1	0	0	0	0	0	0	0
6	0	0	0	0	1	0	0	0	1	1	0	0
7	0	0	0	1	0	0	1	0	1	0	0	0
8	0	0	0	1	0	0	1	1	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0

Example C – C(1,2)

in/out	0	1	2	3	4	5	6	7	8	9	10	11
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	1	1	1	1	1	0	0	0
2	0	1	0	0	1	1	0	0	0	1	0	0
3	0	1	0	0	1	1	1	1	1	0	1	1
4	0	1	1	1	0	1	1	1	1	1	0	0
5	0	1	1	1	1	0	1	1	1	1	0	0
6	0	1	0	1	1	1	0	1	1	1	1	1
7	0	1	0	1	1	1	1	0	1	0	1	1
8	0	1	0	1	1	1	1	1	0	1	1	0
9	0	1	1	1	1	1	1	1	1	0	0	0
10	0	0	0	1	0	0	1	1	1	0	1	1
11	0	0	0	1	0	0	1	1	1	0	1	0

RESULTS CONTINUED

Example C.

Example C – C(1,3)

in/out	0	1	2	3	4	5	6	7	8	9	10	11
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	1	1	1	1	1	0	0	0
2	0	1	0	0	1	1	0	0	0	1	0	0
3	0	1	0	0	1	1	1	1	1	0	1	1
4	0	1	1	1	0	1	1	1	1	1	0	0
5	0	1	1	1	1	0	1	1	1	1	0	0
6	0	1	0	1	1	1	0	1	1	0	1	1
7	0	1	0	1	1	1	1	0	1	0	1	1
8	0	1	0	1	1	1	1	1	0	1	1	0
9	0	1	1	1	1	1	1	1	1	0	0	0
10	0	0	0	1	0	0	1	1	1	0	1	1
11	0	0	0	1	0	0	1	1	1	0	1	0

Example C – C(2,2)

in/out	0	1	2	3	4	5	6	7	8	9	10	11
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	1	1	1	1	1	0	0	0
2	0	1	0	0	1	1	0	0	0	1	0	0
3	0	1	0	0	1	1	1	1	1	0	1	1
4	0	1	1	1	0	1	1	1	1	1	0	0
5	0	1	1	1	1	0	1	1	1	1	0	0
6	0	1	0	1	1	1	0	1	1	1	1	1
7	0	1	0	1	1	1	1	0	1	0	1	1
8	0	1	0	1	1	1	1	1	0	1	1	0
9	0	1	1	0	1	1	0	0	0	0	0	0
10	0	0	0	1	0	0	1	1	1	0	0	0
11	0	0	0	1	0	0	1	1	1	0	0	0

CONCLUSION

The competition Graphical User Interface allows the user to quickly create their desired competition matrix.

The program can keep track of whether an edge exists or not rather then the user. This makes it easier to work with more complex graphs, such as Example 3 which deals with twelve different species.

By applying the user-friendly GUI to the predator-prey research this allows finding relationships between predator-prey a much faster process.

REFERENCES

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[2] Midge Cozzens, Nancy Crisler, Randi Rotjan, and Tom Fleetwood. Biomath: The biology and mathematics of food webs, September 2009.

[3] R. D. Dutton and R. C. Brigham. A characterization of competition graphs. Discrete Applied Mathematics, 1983.

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