Data Assimilation for Fluid Dynamic Models: Finding Flow Paths of an Object Through Water

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### Motivation
Given observational data taken from the trajectory of an instrument flowing in an unknown velocity field, can we recreate the velocity field inducing this motion?

### Methods
Data Assimilation is a process that merges observational data with a mathematical model.

### Strategy
- **Strategy Continued**
  - **1D Sampling Example:** Using Eulerian data assimilation where the observations are fixed. Assume a steady flow; velocity does not change with time.
    - Low degree polynomial fit of data for 1st proposal
  - Prior draws from frequency space, random coefficients \(a_n\) and \(b_n\) of Fourier Series
  - Accept proposal \(v(x)\) from prior draw if it better fits the observations, else repeat initial proposal.

#### Results: Posterior Sampling
- A graph of the area between the 5th and 95th percentile of the posterior distribution and the mean of the distribution.
- A graph showing the posterior distribution conforms to the truth only in the section where observations are available.

#### Results: Posterior Sampling Continued
- The Eulerian 1D sampling performs according to strategy expectations. To achieve the goal of recreating a velocity field from given observational data, there remains the following current & future work:
  - Adapt to Lagrangian data-update likelihood
  - Adapt the strategy to 2D sampling
  - Apply the strategy to data from Dr. Ani Hsieh’s lab

### Selected References
2. Elke Thönnes. Lecture Notes on Monte Carlo Methods University of Warwick.

### Fourier Series (FS)
- Most functions can be described as a series of sines and cosines.

\[
f(x) = \sum_{n=1}^{\infty} a_n \cos(2\pi nx) + b_n \sin(2\pi nx)
\]

**Observe Velocity Field**
**Prior**
**Data**
**Fourier Series**
**MCMC**

**Posterior**

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