Properties of a Food Web with a Complete (1,2)-Step Competition Graph

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Introduction

Food webs are used by ecologists to study the transfer of energy within a given ecosystem. These food webs can be modeled mathematically by acyclic digraphs. In 1968 Cohen introduced the competition graph of a digraph as a method with which ecologists and mathematicians could used a graph on the same vertex set as the digraph modeling the food web to study the relationships between different species in the food web.

In 2001, Factor and Merz extended the conditions of a competition graph to introduce the (1,2)-step competition graph. A (1,2)-step competition graph shows relationships between species which interact indirectly as well as the more direct relationships seen in the competition graph.

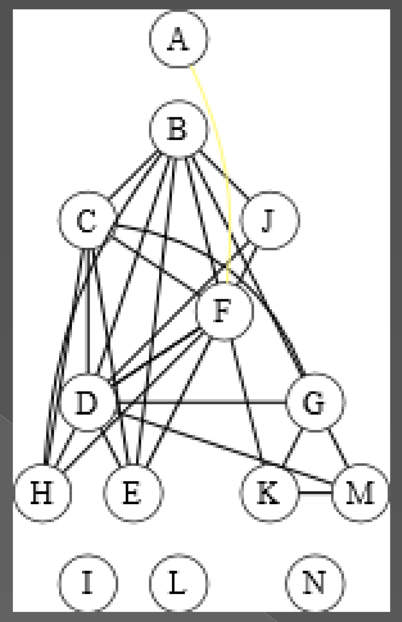
The competition and (1,2)-step competition graphs are used to help in the prediction of how an ecosystem would feel the effects of a species addition to or removal from the ecosystem in question.

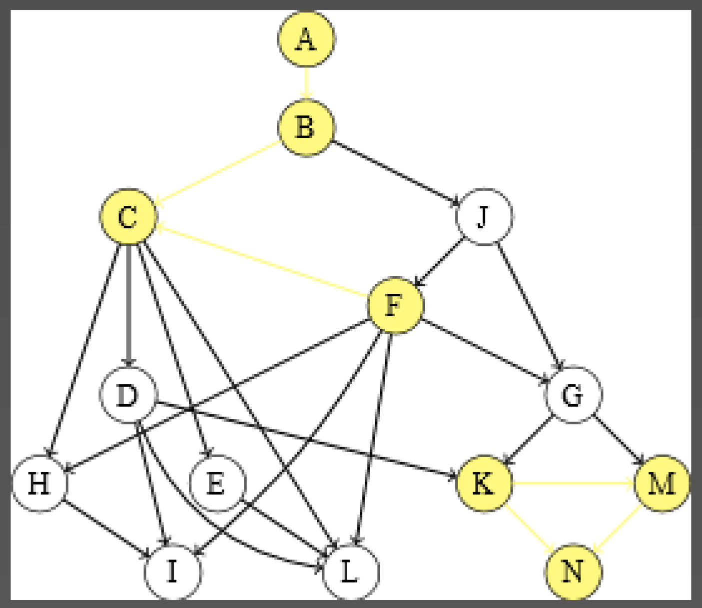
Goal

Discover the properties of real food webs which correspond to (1,2)-step competition graphs which are complete on all non-basal vertices

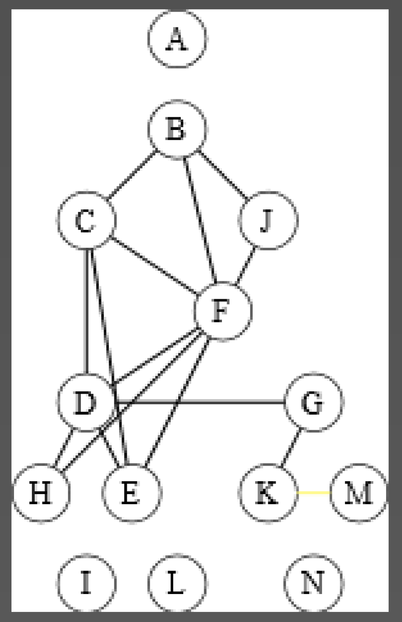
Methodology

- ❖ Use the properties of a (1,2)-step competition graph to inform an exploration of the digraphs which generate complete components
- ❖ Look for patterns in the acyclic digraphs which generate (1,2)-step competition graphs with complete components to see if the patterns reveal a construction for such digraphs
- ❖ Use the properties of real food webs to limit the properties of the digraphs examined as well as to demonstrate patterns which arise in real world applications





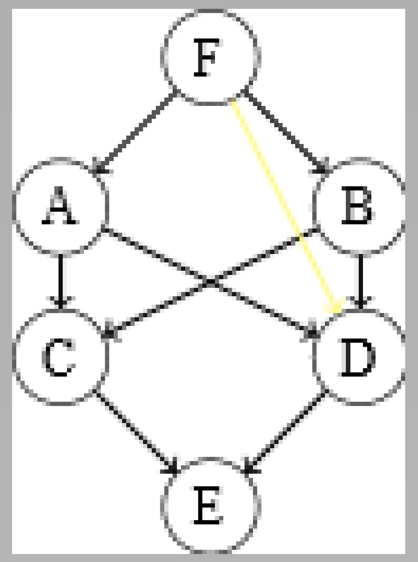
A Sample Food Web of an Oceanic Ecosystem with its Competition and (1,2)-Step Competition Graphs

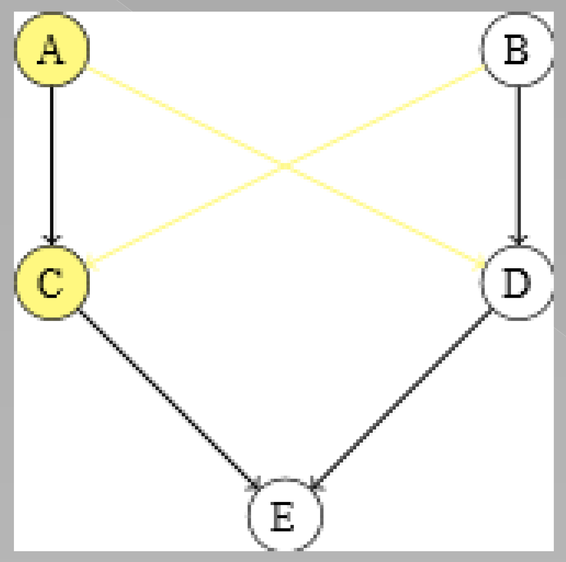


Results

Lemma 1: for all acyclic digraphs, there exists at least one vertex in V(D) which contains only basal vertices in its outset

Theorem 1: If the (1,2)-step competition graph of an acyclic digraph is complete on all non-basal vertices, then each vertex which is not a basal vertex is connected to some basal vertex by a path of length no greater than two





Lemma 2: If the (1-2)-step competition graph of an acyclic digraph is complete on all non-basal vertices, all herbivores must directly compete with all other herbivores

Lemma 3: If there is a vertex with a minimum path length greater than one which has an outset of size one in an acyclic digraph, then the (1,2)-step competition graph of the acyclic digraph is not complete on all non-basal vertices

Theorem 2: If the (1,2)-step competition graph of an acyclic digraph is complete on all non-basal vertices, then all vertices which have a minimum path length of two to a basal vertex also have a second path of maximum length two to the same basal vertex

Future Work

- ❖ Look further into limiting the digraphs examined by applying restrictions based on the properties of practical food webs
- ❖ Find a necessary condition which also has sufficiency for all the non basal species of a (1,2)-step competition graph to be complete
- ❖ Create an algorithm to construct the acyclic digraphs which represent food webs which generate (1,2)-step competition graphs that are complete on all non basal vertices

References

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Acknowledgements

A big thank you to Dr. Factor for al her help an support throughout this project, as well as to Nate Sponberg, my fellow student researcher who was always willing to let me bounce ideas, no matte how nonsensical, off him. I would also like to thank the MSCS department at Marquette University for their hospitality, and the NSF for their support. This work was funded by grant CCF-106304