



# Generalizing and Justifying: Pre-Service K-8 Teachers' Strategies and Representations

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## Introduction

- Inductive Reasoning (i.e. reasoning from specific cases to the general) is an important way of mathematical thinking.
- Multiple ways of thinking about mathematical concepts provide evidence of deeper understanding

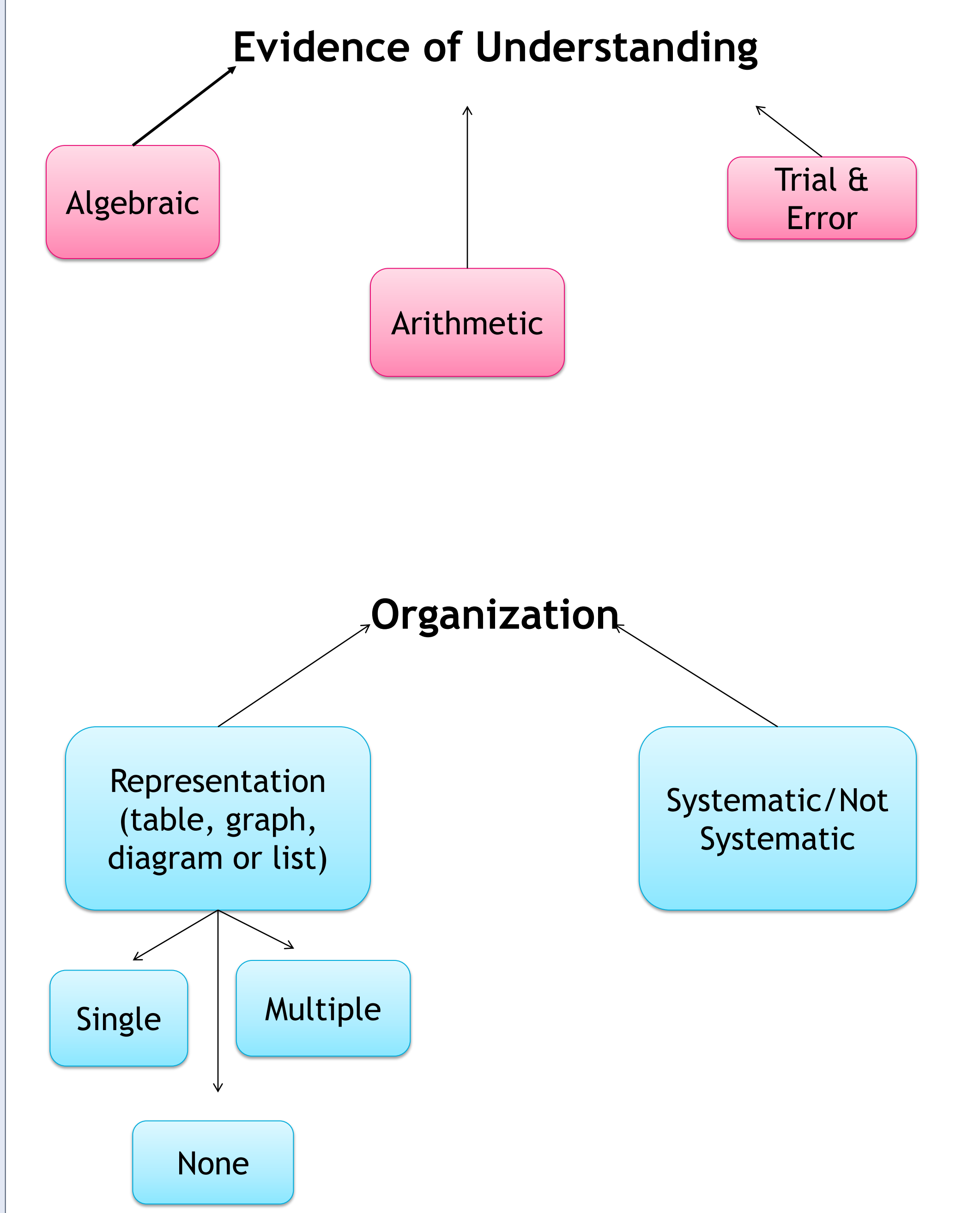
## Objectives

- To identify and describe pre-service K-8 teachers' processes of generalizing and justifying:
  - Strategies
  - Representations
  - Use of visual/structural and numerical information

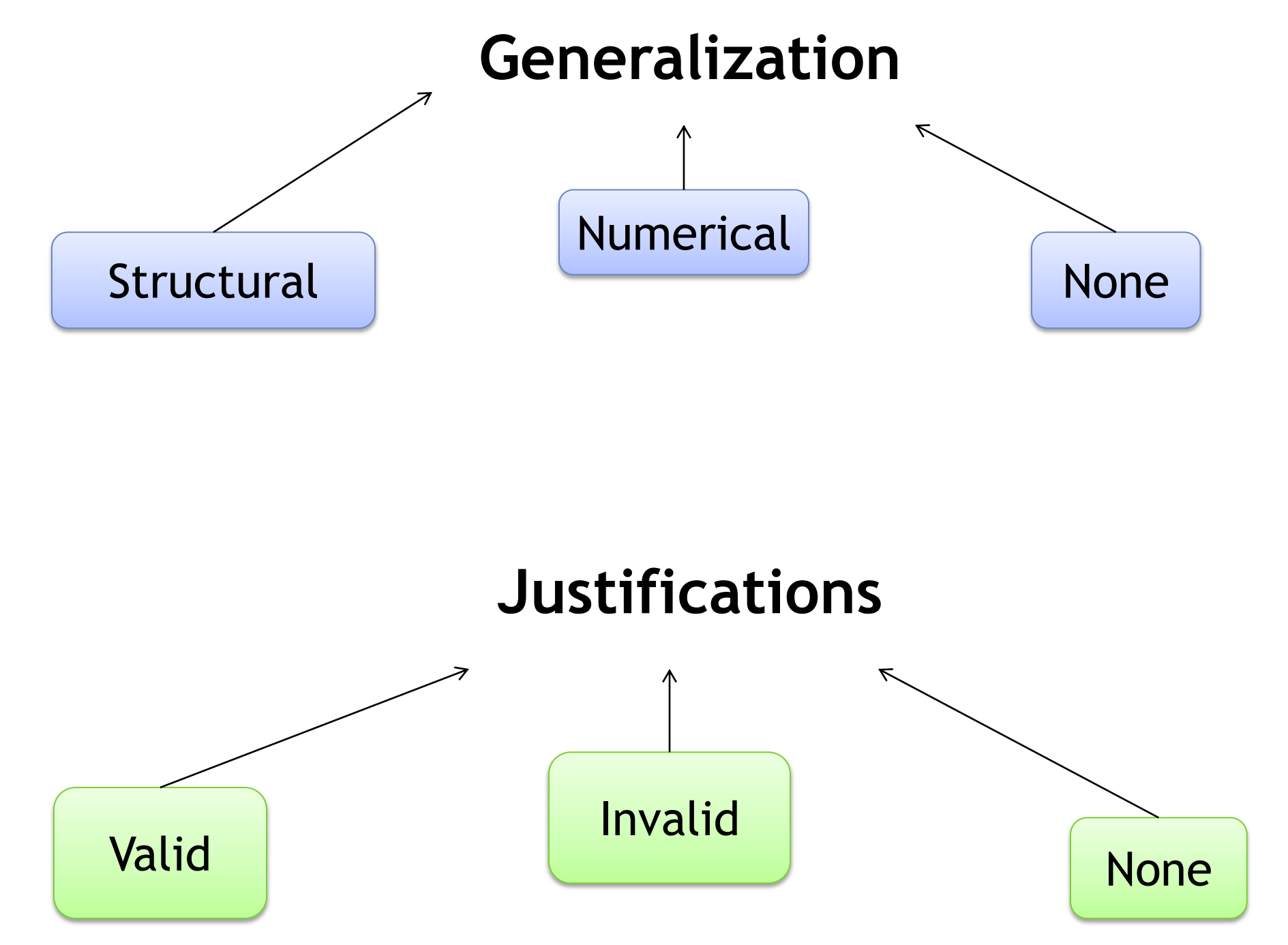
## Methodology

- 17 Pre-Service Teachers
- 184 written solutions to pattern finding-tasks
- Qualitative analysis of solutions using specific rubric

## Data Analysis



## Data Analysis cont....



## Sample: Type of Work Reviewed

**Black and White Tiles Activity**

Mary uses black and white tiles to make figures like the ones presented below:

Figure 1: 5 black, 16 white, 21 total  
Figure 2: 9 black, 24 white, 33 total  
Figure 3: 13 black, 32 white, 45 total  
Figure 4: 17 black, 40 white, 57 total

Figure	Black	White	Total
1	5	16	21
2	9	24	33
3	13	32	45

**Systematic diagram and table to organize**

**Use of algebra and arithmetic to solve**

$$T_n = (4n+1) + (8n+12-4) = 12n+9$$

**Use of algebra and arithmetic to solve**

$$T_n = 12n+9$$

**Generalizations made with numbers only were the most frequently observed**

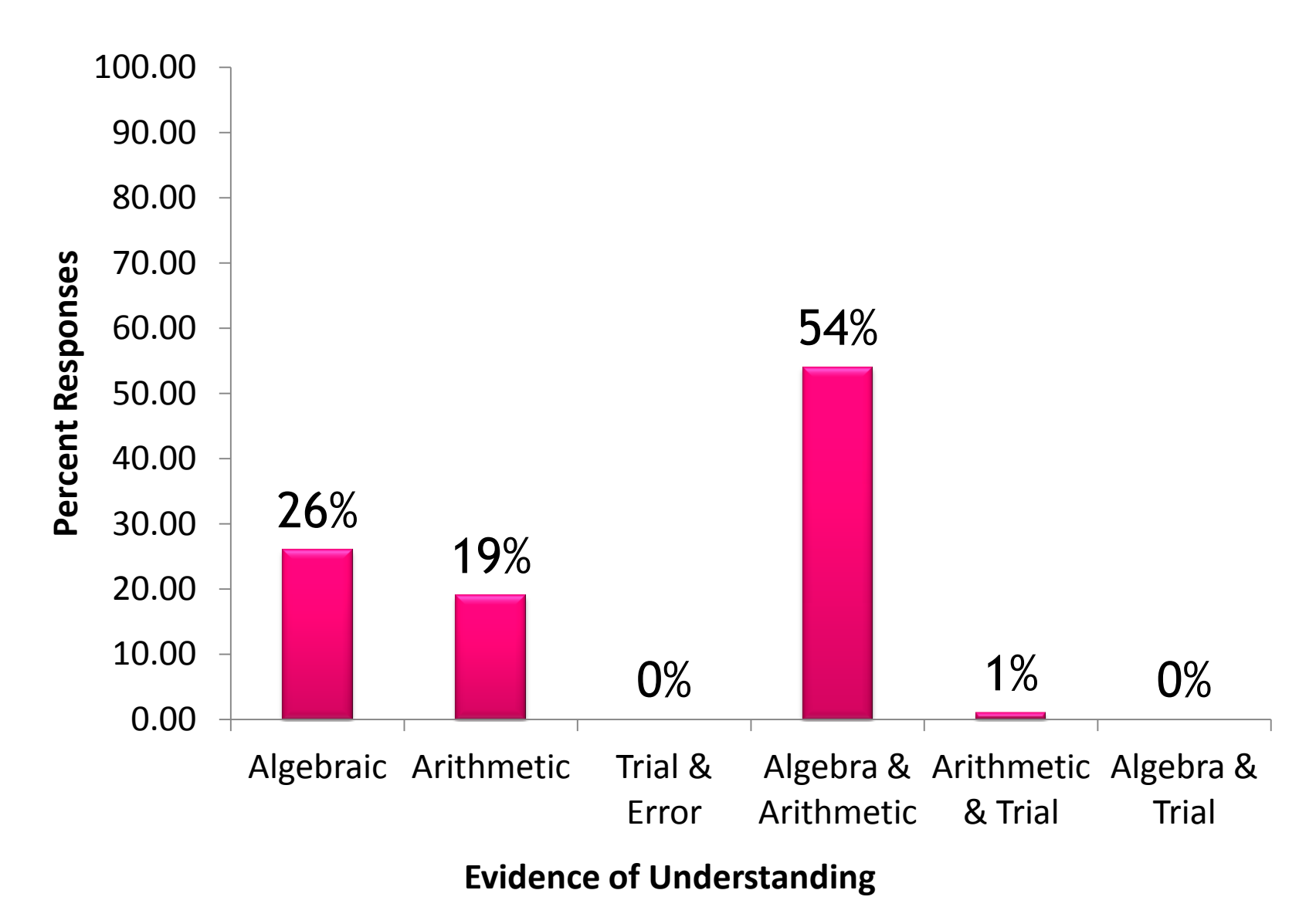
- Simplified into explicit formula
- Linked numbers to structure
- Extended to ALL cases

**Gave a valid justification in explanation**

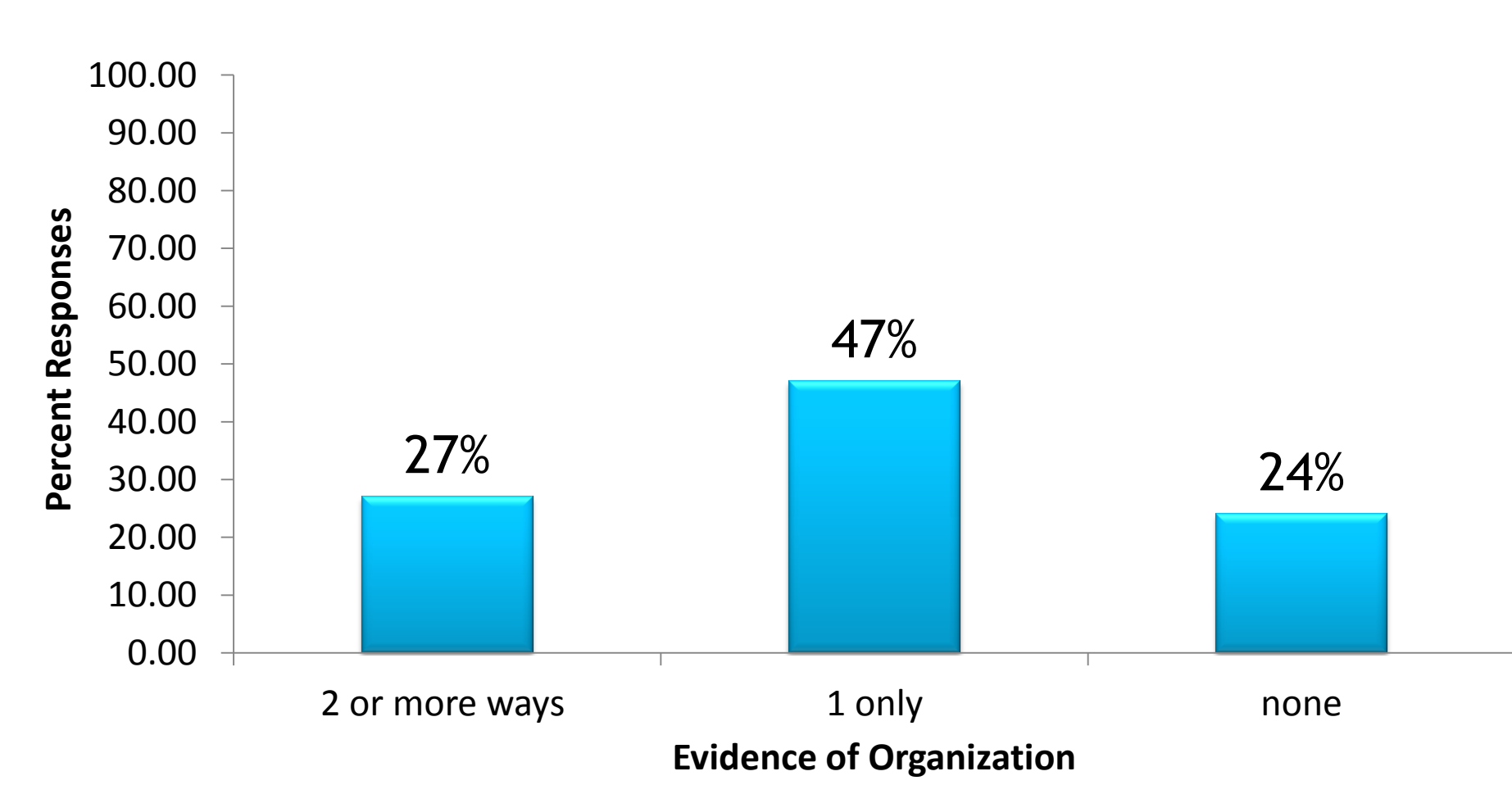
This rule adds together the number of black squares with the number of white squares. There is always a black square in the middle of the figure. (1). The black squares then extend in 4 directions, containing the figure number of squares in each direction (4n). The number of white squares surrounding each of the 4 black square extensions is double the figure number or black squares extended from the middle to account for the sides and then always 3 more to account for the end of each extension, multiplied by 4 because the black squares extend in 4 directions. 4 is then subtracted from this figure to account for the 4 inner corners that get counted more than once. (I simplified this equation to 12n+9 to solve for different figure numbers to make things easier to work with)

## Results and Discussion

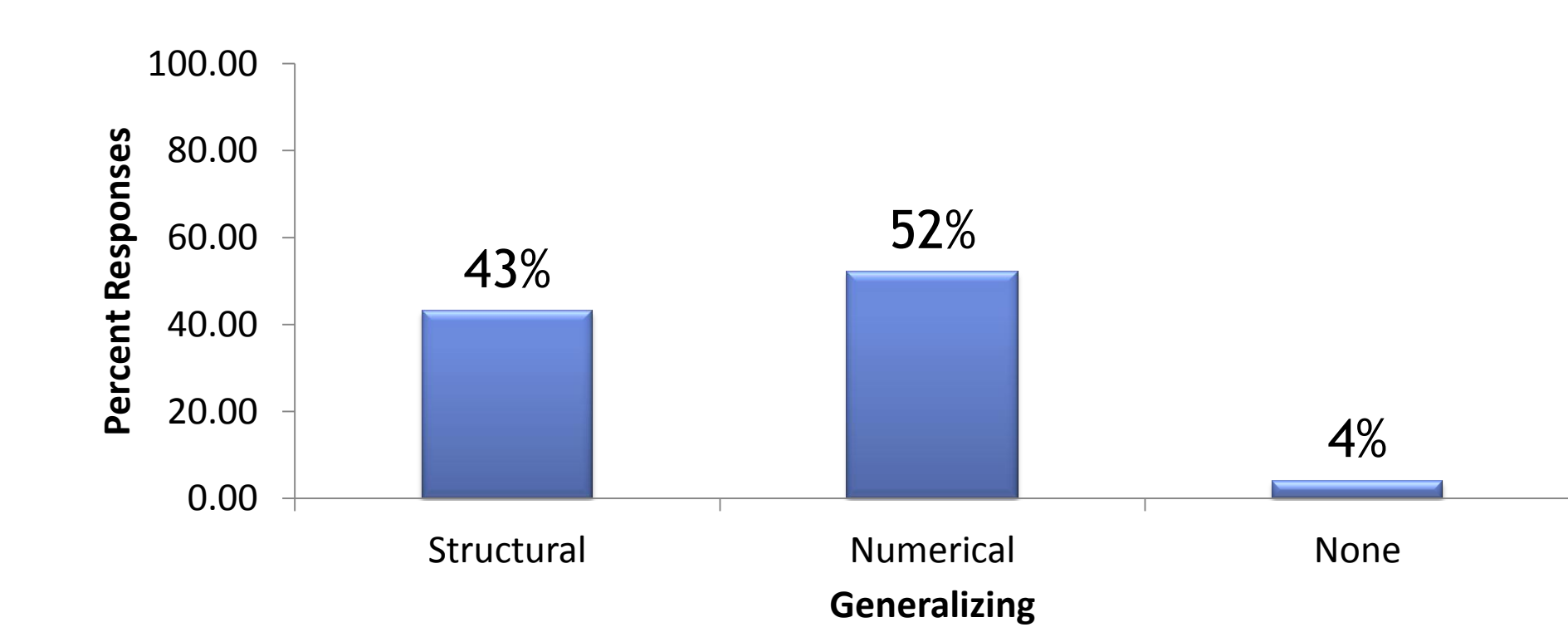
➤ Algebra & Arithmetic together were the most frequently observed



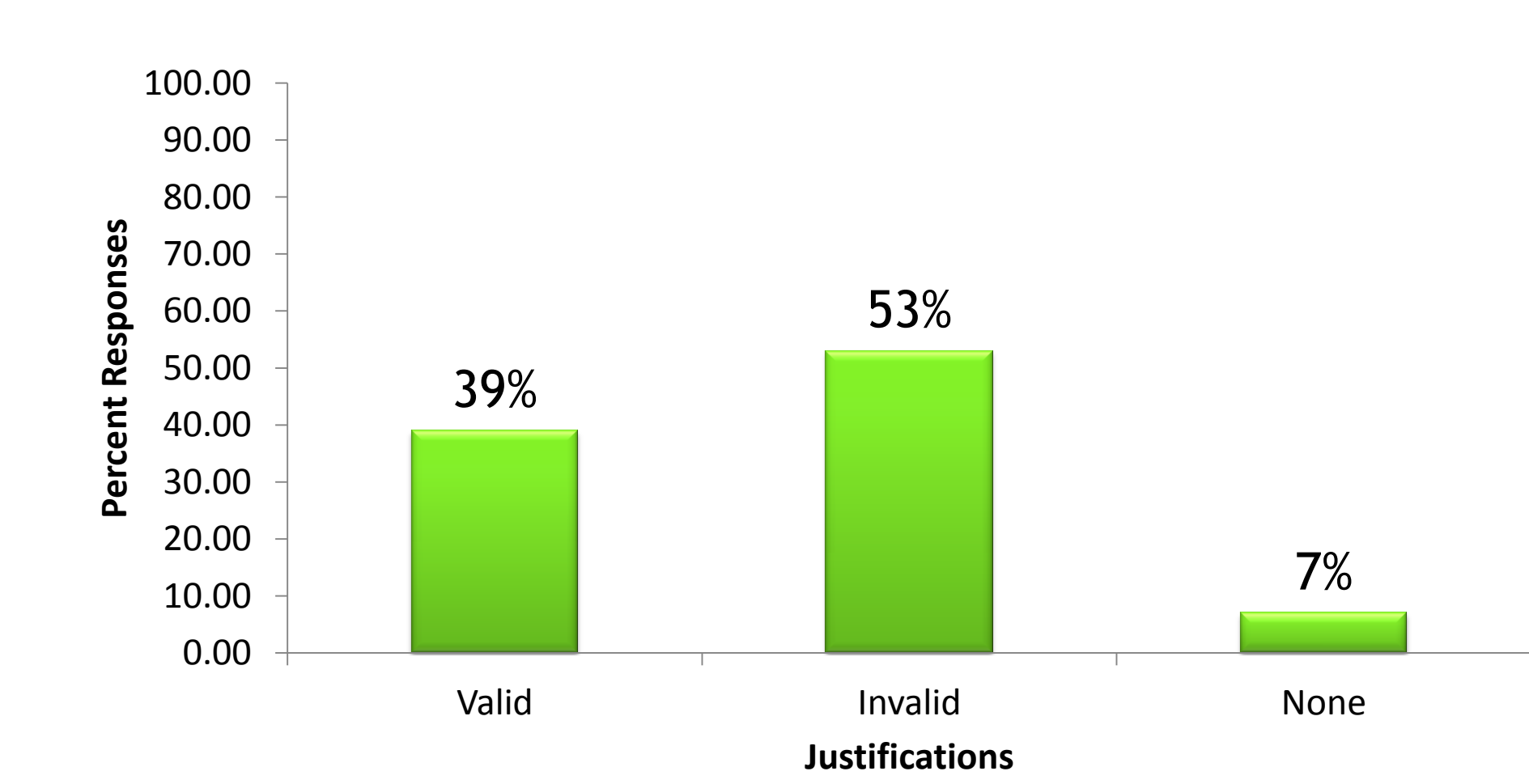
➤ The use of one method of organization was most frequently observed



➤ Generalizations made with numbers only were the most frequently observed



➤ Invalid justifications were most frequently observed



## Conclusions

- Pre-service teachers' can solve pattern finding problems using various strategies
- They use at least one method of organization during problem-solving process
- Although generalizations are provided, the majority are based on numbers only; no links are made comparing numbers to structure
- Many justifications are made but most are invalid

## Bibliography

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