Calculating the Effect of Competition on Survivability

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INTRODUCTION

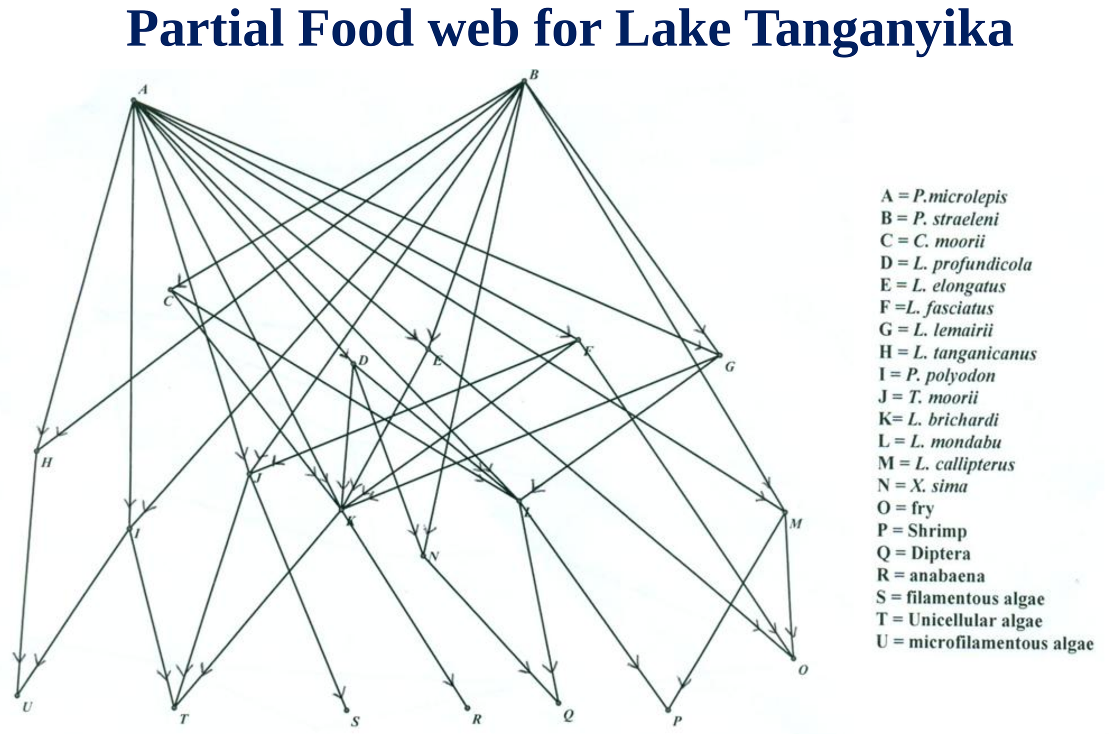
Cohen (1968) introduced the competition graph of a digraph to study food web patterns and relationships between species. The vertices represent the various species and directed edges known as "arcs" represent the predator-prey relationship between the species. This has been studied extensively. Two extensions to the theory include Factor and Merz (2010), who introduced the (1,2)-step competition graph of a digraph by first examining tournaments and Sano (2007),who introduced the weighted competition graph.

In this research, we find the (1,2)-step competition graphs and weighted competition graphs of actual food webs (acyclic digraphs) and specifically for the ecosystem of Lake Tanganyika. We also introduce the percentage-weighted competition digraph to help determine the effect of competition on survivability of each species in the ecosystem.

OBJECTIVES

- ❖ Extend the definition of a weighted competition graph to a weighted (1,2)-step competition graph
- ❖ Introduce the percentage-weighted competition graph

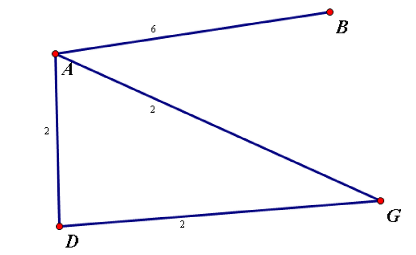
Partial Food web for Lake Tanganyika



METHOD AND DEFINITIONS

❖ **Create the competition graph of a food web D and compute edge weights**

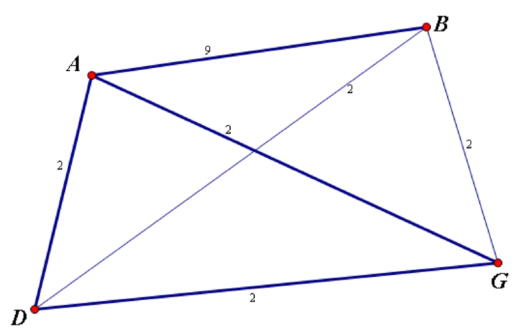
The competition graph has edges between species that share a common prey. The weight is the number of common prey that A and B share.



Subgraph of $C_w(D)$ for Lake Tanganyika

❖ **Create the (1,2)-step competition graph of a food web, D, and find $C_{w(1,2)}(D)$, the weighted (1,2)-step competition graph of D**

The (1,2)-step competition graph, $C_{1,2}(D)$ is a graph on the same vertex set as D, where xy is an edge if and only if there exists a vertex $z \neq x, y$ such that either the shortest distance from x to z uses one arc & the shortest distance from y to z (without going through x) uses at most 2 arcs or vice versa.



Subgraph of $C_{w(1,2)}(D)$ for Lake Tanganyika

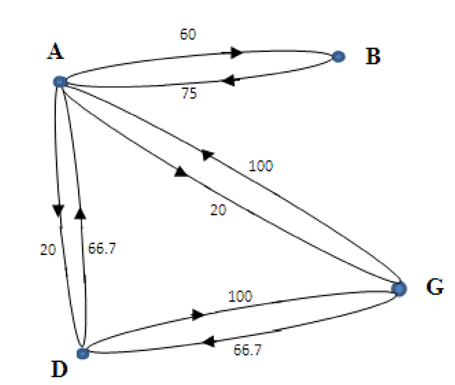
❖**The percentage-weighted competition digraph of D, P_w C(D)**

P_w C (D) is an arc-weighted digraph derived from C(D) such that

$$P_w(u,v) = \left(\frac{w(uv)}{d^+(u)}\right) * 100$$

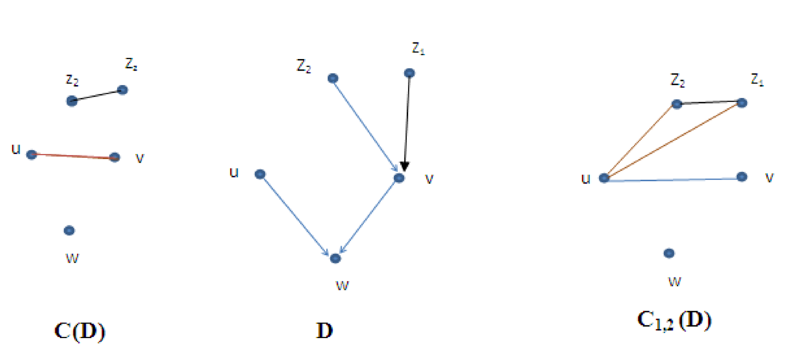
$$\text{and } P_w(v,u) = \left(\frac{w(uv)}{d^+(v)}\right) * 100$$

Where w(uv) is the weight of edge uv in C(D) and $d^+(u)$ is the number of species u preys upon.

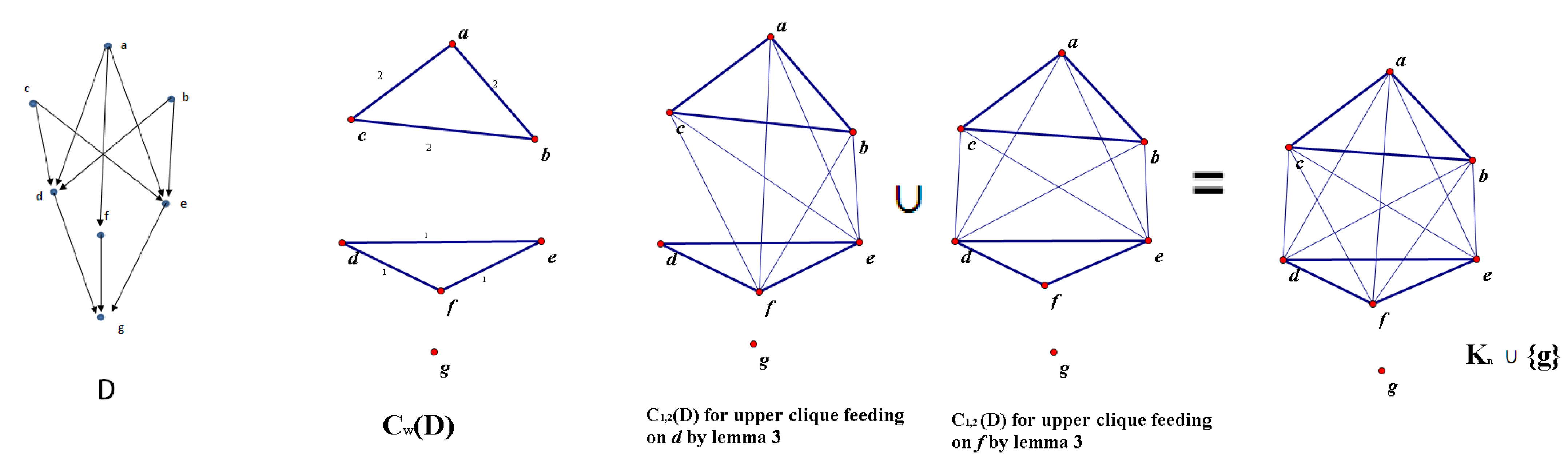


RESULTS

Lemma 3: Let uv be an edge in C (D). For all $z \neq u$ that are in $N^-(v)$ (the species that prey upon v) in D, *uz* is an edge in $C_{1,2}(D)$.



Theorem: Let $C_w(D)$ be the weighted graph comprised of k isolated vertices (primary producers) and 2 disjoint cliques (S_i, S_j) of consecutive shortest-path trophic levels, where $|V(S_i)| = n_i$, $|V(S_j)| = n_j$ and $n_i + n_j = n$.
If w(e) >1 for the uppermost level in $C_w(D)$, then $C_{1,2}(D)$ is $K_n \cup$ {k isolated vertices}.



Theorem: Let xy be an edge in C(D). There is an edge between vertices x and y in the P_w C(D) if an only if the degree of x equals the degree of y in D.

Application

The total out-degree of a species in a P_w C(D) digraph is proportional to the effect of competition on it survivability.

CONCLUSIONS & FUTURE WORK

❖ A relationship exists between $C_w(D)$ & $C_{1,2}(D)$.

❖P_w C(D) shows the effect of a vertex removal on survival of other species in the food web. The percentage-weight of the in-degree of the removed vertex will determine the magnitude of the positive effect on each of the adjacent vertices.

FUTURE WORK:

❖Extension of $P_w(D)$ to $C_{1,2}(D)$ and the m-step competition graph.

❖ Apply to other applications such as networking.

❖Characterize with the adjacency matrices of the digraphs and graphs.

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