Predicting Flows with Filters and Ergodicity
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Introduction
Our goal is to create a means to more accurately predict flow trajectories based on partial state observation.
In order to accomplish this task, we are looking into modifying a particle filter to make it more robust by adding criteria to the filter, and checking the consistency with ergodic theory.
Jack has been working with Dr. Scott on ergodic theory
Gabe has been working with Dr. Spiller on particle filters

Ergodicity

What is Ergodicity?
Samples the entire space
Ergodic: if a map is said to have ergodicity, we say that it is ergodic

Graph of defect vs scale for computer generated answers and theoretical answers

Tested two 2-dimensional maps:
Invariant Circles
y doesn’t change
x moves across space from left to right

Nonergodic Island Map
y greater than .5, map is non-ergodic
y less than .5, map is ergodic

Graph of defect vs scale for computer generated answers and theoretical answers

Nonergodic Island Map
Defect vs Scale

What is a particle filter?
A way of combining model predictions and observation to get an estimate of a system’s state
We are interested in systems where particle trajectories follow complicated nonlinear paths
How does it work?
1. Begin with distribution (particle cloud) of initial conditions
2. This distribution is a set of discrete state values which are weighted in order to reflect uncertainty in the initial conditions
3. Move particles forward in time
4. Resequence according to new positions relative to an observation i.e. particles that are closer to the observation get higher weights
5. Repeat process for subsequent observations

Use this method to estimate flow paths
This method can break down if few or no particles in the cloud are near an observation value
Models can be expensive to compute, so we would like to use as few particles as possible to accurately filter
Monitoring the ergodic defect may indicate instances when a more robust filter is needed

Shallow Water Equations
What are shallow water equations?
A set of partial differential equations for u (horizontal velocity), v (vertical velocity), and h (height) to describe flow trajectories
= u, = v.
Produces flow fields such as the one below

The flow field changes with time and therefore do the trajectories
Using a particle filter, we can determine the paths of trajectories in the flow fields, and track the movement of the system

Future Work
Combining filters and ergodicity work
Help to determine when a filter fails
Increase accuracy of filter
Better filter monitoring
Development of a more sophisticated filter

References