



Pre-service Teachers' Algebraic Habits of Mind



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Motivation

In recent years mathematics educators, researchers, and policymakers have advocated for the introduction of algebra related concepts into the K-8 mathematics curriculum. The goal was to provide the K-8 students with access to algebraic ideas by helping them develop new ways of thinking by engaging them in analyzing quantitative situations in a relational way. Thus the focus of K-8 algebra was algebraic thinking in contrast to traditional algebra based courses that emphasize symbolic manipulations and solving equations.

To successfully introduce algebra-based concepts in the middle grades teachers need to understand the ideas behind algebraic thinking. They need the ability to think algebraically, they need the ability to recognize opportunities to engage middle school students in different features of algebraic thinking, and they need the ability to elicit and interpret features of algebraic thinking in the work of students.

The emphasis given to algebra at the K-8 level requires teacher education programs to strengthen their efforts to equip prospective teachers with necessary skills in order to effectively teach algebra-based concepts in their future classrooms.

Algebraic Thinking

Algebraic thinking is broadly defined as habits of mind (Cuoco, Goldberg & Mark, 1996) and described as useful ways of thinking about mathematical content. In this research we focused on selected features of algebraic habits of mind described by Driscoll (1999).

Feature of Algebraic Habits of Mind	Description of Thinking Exemplified
Organizing Information patterns	Ability to organize information in ways useful for uncovering
Predicting Patterns	Ability to notice a rule at work and make sense of how a rule works
Chunking Information	Ability to look for repeating chunks in information about a pattern
Different Representations	Ability to think about and try different representations of the problem in order to uncover different information about the problem.
Describing a Rule	Ability to describe steps of a procedure or a rule
Describing Change	Ability to describe change in a process or a relationship
Justifying a Rule	Ability to justify why a rule works for any number
Doing/Undoing	Ability to find input from output and the ability to work steps of a rule or procedure backwards

Research Focus

- This research examined the pre-service middle school teachers' knowledge of algebraic thinking by
 - identifying strength of pre-service teachers algebraic thinking, and
 - assessing how well the pre-service teachers were able to elicit and interpret the algebraic thinking of middle school students.
- The data for the study were collected from an integrated mathematics and field experience course for pre-service middle school teachers designed to strengthen PSTs' knowledge of algebraic thinking. Eighteen pre-service teachers participated in the study. 130 tasks were analyzed.

Data Analysis

- Each task was analyzed in three steps.
 - Specific algebraic habits of mind elicited by the task were identified and documented.
 - Pre-service teachers' solutions to each task were qualitatively examined to identify features of algebraic habits of mind evident in pre-service teacher work.
 - Pre-service teachers' performance on each identified feature of algebraic habits of mind was assessed on a 3 point scale (3- proficient, 2- developing, 1- no understanding).
- The strengths and patterns of the PSTs' algebraic thinking were identified by qualitatively analyzing the extent to which participants' solutions and explanations, found in class assignments and tests (130 tasks analyzed), exhibited various features of algebraic thinking.
- Average performance scores across all tasks were used to examine patterns in pre-service teachers' performance on different features of algebraic thinking.

Results

- The pre-service teachers' ability to demonstrate different features of algebraic habits of mind varied among the features examined. Among the examined features of algebraic habits of mind, the pre-service teachers' ability to justify was by far the weakest one and their ability to work backwards was the strongest one.
- The overall mean performance score for demonstrating algebraic habits mind was $M = 2.5$, $SD = 0.16$.
- Overall, pre-service teachers had difficulties eliciting and interpreting the algebraic thinking of students. Pre-service teachers were most successful in noticing and interpreting how students organize information ($M = 2.61$), chunk information ($M = 2.55$) and justifying a rule ($M = 2.55$). The weakest performance was noted for interpreting how students describe change ($M = 2.27$) and how they predict patterns ($M = 2.27$).
- A strong relationship was found between the pre-service teachers' ability to demonstrate features of algebraic habits of mind and their ability to interpret features of algebraic habits of mind in the work of students.

Conclusions

- Pre-service teachers should be engaged in activities that support their own algebraic thinking as well as in eliciting and reflecting on the algebraic thinking of K-8 level students.
- Understanding the complexity of pre-service teachers' knowledge of algebraic thinking can inform the practice of preparing teachers to foster algebraic thinking in their own classrooms.
- Strong links shown between pre-service teachers' own algebraic thinking and their ability to elicit and interpret the thinking of students show the importance of designing teacher education programs that focus on developing both aspects of teachers' knowledge of algebraic thinking.

References

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