Instructions: Read carefully through the whole exam first and plan your time. Note the relative weight of each question and part (as a percentage of the score for the whole exam). The total points is 100 (your grade will be the percentage of your answers that are correct).

This exam is closed book, closed notes. You may not refer to any book or other materials.

You have one hour to complete all four (4) questions. Write your answers on this paper (use both sides if necessary).

Name:
Student Number:
Signature
1. (Regular expressions; 25%) Write regular expressions that define the strings recognized by the following finite automata:

(a) (5%)

Answer:
\[(ab)^*(\varepsilon \mid a(bb)^*)\]

(b) (10%)

Answer:
\[(ab \mid aba)^*\]

(c) (10%)

Answer:
\[(aaa)^* \mid aa(aa)^*\]
2. (Finite automata; 25%) Consider the following regular expression:

\[(ab | aab | aba)^*\]

(a) (10%) Draw a simple NFA for this expression (i.e., without the extra \(\varepsilon\)-transitions that result from the more mechanical construction that was described in class).

Answer:

```
2  b  a  4
  a  b
1  3
  a  a
5  b  6
```

(b) (10%) Using the subset construction, convert the NFA into a DFA.

Answer:

```
A = \{1\}  a  b
B = \{2,3,5\}  A  B  C  D
C = \{4\}  B  C  A
D = \{1,6\}  C  D
E = \{1,2,3,5\}  D  E
F = \{2,3,4,5\}  E  F  D
                  F  C  D
```

A is the start state.
A, D, and E are final states.

(c) (5%) Try to understand how the DFA constructed in part 2b operates. Based on your understanding, optimize the DFA as necessary to minimize states.

Answer:

States B and F are equivalent, yielding:

```
a  b
A  B
B  C  D
C  A
D  E
E  B  D
```
3. (Context-free grammars; 25%) Consider the grammar

\[ S \rightarrow aSbS \\
S \rightarrow bSaS \\
S \rightarrow \varepsilon \]

(a) (5%) Show that this grammar is ambiguous by giving two different leftmost derivations for the sentence \( abab \).

Answer:

\[
\begin{align*}
S \Rightarrow_{lm} & aSbS \quad S \Rightarrow_{lm} aSbS \\
& \Rightarrow_{lm} abSaSbS \quad \Rightarrow_{lm} a[\varepsilon]bS \\
& \Rightarrow_{lm} ab[\varepsilon]aSbS \quad \Rightarrow_{lm} a[\varepsilon]baSbS \\
& \Rightarrow_{lm} ab[\varepsilon]a[\varepsilon]bS \quad \Rightarrow_{lm} a[\varepsilon]ba[\varepsilon]bS \\
& \Rightarrow_{lm} ab[\varepsilon][a[\varepsilon][b[\varepsilon] \quad \Rightarrow_{lm} a[\varepsilon]ba[\varepsilon][b[\varepsilon]
\end{align*}
\]

(b) (5%) Give the corresponding rightmost derivations for \( abab \).

Answer:

\[
\begin{align*}
S \Rightarrow_{rm} & aSbS \quad S \Rightarrow_{rm} aSbS \\
& \Rightarrow_{rm} aSb[\varepsilon] \quad \Rightarrow_{rm} aSbaSbS \\
& \Rightarrow_{rm} abSaSb[\varepsilon] \quad \Rightarrow_{rm} aSbaSb[\varepsilon] \\
& \Rightarrow_{rm} abSa[\varepsilon][b[\varepsilon] \quad \Rightarrow_{rm} aSba[\varepsilon][b[\varepsilon] \\
& \Rightarrow_{rm} ab[\varepsilon][a[\varepsilon][b[\varepsilon] \Rightarrow_{rm} a[\varepsilon]ba[\varepsilon][b[\varepsilon]
\end{align*}
\]

(c) (10%) Draw the corresponding parse trees for \( abab \).

Answer:

(d) (5%) What language does this grammar generate?

Answer:

Strings with an equal number of \( a \)s and \( b \)s.
4. (LL parsing; 25%) Consider the grammar

\[
S \rightarrow U; W \\
U \rightarrow Ua \\
U \rightarrow \epsilon \\
W \rightarrow bW' \\
W' \rightarrow W \\
W' \rightarrow \epsilon
\]

(a) (20%) Derive the LL(1) parse table for the language generated by the grammar. [Hint: You may have to transform the grammar first in order to do so.] Answer:

First, left-factor and eliminate left-recursion:

\[
S \rightarrow U; W \\
U \rightarrow aU \\
U \rightarrow \epsilon \\
W \rightarrow bW' \\
W' \rightarrow W \\
W' \rightarrow \epsilon
\]

<table>
<thead>
<tr>
<th>FIRST</th>
<th>FOLLOW</th>
<th>a</th>
<th>b</th>
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</tr>
<tr>
<td>W'</td>
<td>b$ε</td>
<td>$</td>
<td>$W'</td>
<td>$W'</td>
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</tbody>
</table>

(b) (5%) Of the language classes we have discussed in this course (i.e., regular or context-free), what is the smallest category into which this language fits? Explain your answer. Answer:

This language is regular. It can be captured by the following regular expression: $a^*; bb^*$