1. Draw the finite state machines, and write regular expressions that denote the strings recognized by the following finite automata:

(a) $s_1$ is the start state; $s_2$ is the final (accepting) state

```
  a   b
s1  s2
s2  s1
```

(b) $s_1$ is the start state; $s_3$ is the final state

```
  a   b
s1  s2  s3
s2  s3
s3  s3
```

(c) $s_1$ is both the start state and the final state

```
  a   b
s1  s2
s2  s3  s1
s3  s3  s2
```

(d) $s_4$ is both the start state and the final state

```
  a   b   c
s1  s4
s2  s4
s3  s4
s4  s2  s1  s3
```

(e) $s_1$ is the start state; $s_6$ is the final state

```
  a   b
s1  s2  s4
s2  s3  s2
s3  s4  s6
s4  s5  s3
s5  s5  s6
s6  s6  s6
```
2. Write DFA’s that recognize the following languages:

(a) \( \{ w \in \{a, b\}^* : \text{each } a \text{ in } w \text{ is followed by a pair of } b\text{'s } \} \)
(b) \( \{ w \in \{a, b\}^* : w \text{ has no consecutive } a\text{'s } \} \)
(c) \( \{ w \in \{a, b\}^* : w \text{ has an even number of } a\text{'s and an odd number of } b\text{'s } \} \)
(d) \( \{ w \in \{a, b\}^* : w \text{ has } abba \text{ as a substring } \} \)
(e) \( \{ w \in \{a, b\}^* : w \text{ has both } baa \text{ and } abb \text{ as a substring } \} \)

3. Consider the following regular expression:

\( (aab \mid abb)^* \)

(a) As described in class, construct an NFA for this expression.
(b) Using subset construction, convert the NFA into a DFA.
(c) Optimize the DFA to minimize the number of states.