

Nonlinear Filtering for Data Assimilation of Semi-Lagrangian Ocean Glider Data

Adam Mallen

Department of Mathematics, Statistics, and Computer Science

March 8, 2011

Motivation

- ▶ Traditionally ocean researchers use unguided buoys for taking ocean measurements.
- ▶ With these traditional measuring devices, launch points have to be chosen with the hope that the buoys would drift to places of interest.
- ▶ The recent development of ocean “gliders” gives researchers the ability to maneuver measuring devices to places of interest.
- ▶ Gliders are capable of guided 3-D movement through the ocean.

Issues With Ocean Glider Modeling

Partial Observations:

- ▶ We can only observe the location of the tracers/gliders.

Measurement Noise:

- ▶ Location observations occur with error.

Nonlinear Models:

- ▶ Ocean processes are often nonlinear.

Model Noise:

- ▶ Our model dynamics use a stochastic component to handle unresolved factors.

High Dimensionality:

- ▶ Many traditional data assimilation schemes for tracers use only two spatial dimensions, but gliders require 3-D models.

The research problem statement is,

“How can we use noisy and infrequent observations of the gliders’ positions to estimate the effect of ocean dynamics on the future movement of the gliders between observation times?”

Further, we want to do this in an effective and computationally feasible way.

Point-Vortex System

- ▶ It is an idealized nonlinear model of realistic fluid processes that gliders and tracers encounter in the ocean.
- ▶ We give the dynamics a stochastic component for unresolved factors. So, we model the point-vortex system with a nonlinear stochastic differential equation.

$$dx = F(x, t)dt + G(x, t)dW$$

where x is the state variables, W is a standard Weiner process, and F and G are the deterministic and random components of the model, respectively.

- ▶ Most of my research focuses on a model with two vortices, which rotate around each other, and a single tracer which floats around the two vortices according to the fluid flow they generate.

Two Point-Vortex System Example

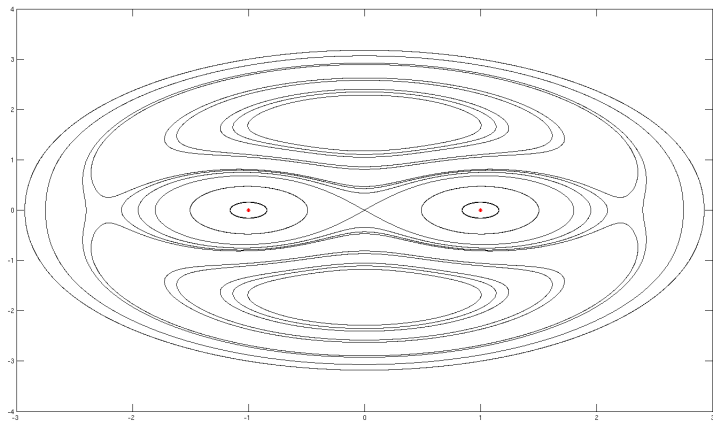


Figure: Sample tracer paths around two vortices of equal strength.

Data Assimilation

- ▶ Data assimilation is the science of using noisy empirical data to improve imperfect model based estimations of a process.
- ▶ Often, we only observe a select number of characteristics of a process, but we want knowledge of unobservable characteristics or the state of measurable characteristics between observation times.
- ▶ In this case, we can only observe the location of the tracer at observation times, and these will contain measurement error. However, we are also interested in the position and strengths of the vortices as well as the tracer locations between observation times.

- ▶ Filtering is a common data assimilation technique used in these kinds of problems.
- ▶ We can think of a probability density of each state variable at each time step and use these for estimates of the system's state.
- ▶ Densities are evolved through time according to the model dynamics and updated at each observation according to the likelihood of yielding the current observation.
- ▶ If the model dynamics are linear and the observation errors are Gaussian, then an application of Bayes rule yields the well known Kalman filter.
- ▶ Kalman filters work well with high dimensional systems, but require a linear system (or a linear approximation of the system).

Particle Filters

- ▶ We can approximate these densities by using particle filters which make no assumptions on the linearity of the system or the distribution of the observation noise.
- ▶ Particle filters use a large number of weighted random samples (particles) as a discrete approximation of these densities.
- ▶ Each particle is evolved through time and then re-weighted at each observation according to the likelihood of the current observation. The re-weighted particles make up the new density at the current observation time.
- ▶ Particle filters work fine on nonlinear problems, but become computationally unwieldy as the state dimension increases.

Particle Filter Example

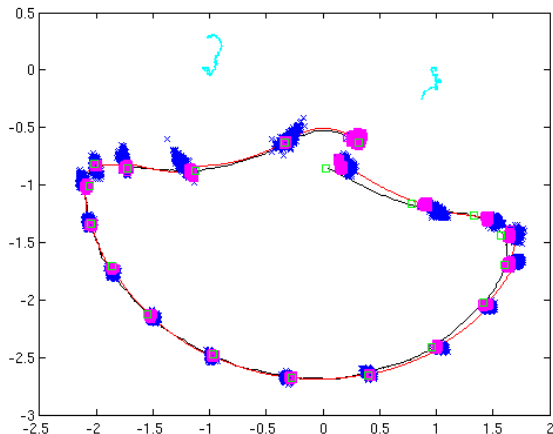


Figure: Sample implementation of a particle filter.

My research plan:

- ▶ (1) I start by reproducing the current data assimilation schemes of tracer data on the 2-D point-vortex system.
- ▶ (2) Next, I will investigate applying these assimilation techniques to the same system with unknown, time-varying, vortex strengths.
- ▶ (3) Then, I will study the problem of assimilating glider data in order to control its guided movement through the same system.
- ▶ (4) Finally, I will translate the work on the 2-D glider control problem to a 3-D system.

Most of my research has been programming the particle filter.
I have taken an iterative approach to building this large program.

- ▶ A typical coding iteration takes between one to two weeks.
- ▶ New features are added to the program incrementally with these iterations.
- ▶ Each iteration involves unit tests for its changes.
- ▶ Many iterations also involve lengthy experiments to either
 - ▶ confirm that code changes have not affected the results
 - ▶ or to measure how the changes have affected the results.

Some example programming iterations have been to

- ▶ Build a stochastic RK4 numerical integration method
- ▶ Implement a particle filter on a simple SDE
- ▶ Implement a particle filter on the point vortex problem
- ▶ Allow simulation to read parameters from an input file
- ▶ Allow an arbitrary number of vortices and tracers in the system
- ▶ Allow unknown (constant or time varying) vortex strengths
- ▶ Run simulations in parallel on Pere cluster.

Questions?

Thanks!

Any Questions... ?